

**ANTOINE D'AUTUME** ECONOMIC POLICY, INNOVATION AND GROWTH

**MURIEL DAL-PONT / DOMINIQUE TORRE / ELISE TOSI** SHORT-TERM DEVIATIONS FROM MONETARY POLICY COMMITMENT IN A MONETARY UNION

**ALESSIO MONETA** IDENTIFICATION OF MONETARY POLICY SHOCKS: A GRAPHICAL CAUSAL APPROACH

**JOÃO SOUSA ANDRADE** RÉGIMES MONÉTAIRES ET THÉORIE QUANTITATIVE DU PRODUIT NOMINAL AU PORTUGAL (1854-1998)

**ANTÓNIO CALEIRO** ECONOMIC POLICIES AND ELECTIONS: A PRINCIPAL-AGENT POINT OF VIEW

**MARIA ADELAIDE DUARTE / MARTA SIMÕES** HUMAN CAPITAL, MECHANISMS OF TECHNOLOGICAL DIFFUSION AND THE ROLE OF TECHNOLOGICAL SHOCKS IN THE SPEED OF DIFFUSION

**PETER RÜHMANN** THE GERMAN LABOUR MARKETS – THE CASE FOR MORE FLEXIBILITY

**PEDRO GODINHO / JOÃO PAULO COSTA** THE USE OF COST AND TIME IN PROJECT DECISION TREES

**J. L. MIRALLES MARCELO / M.ª DEL MAR MIRALLE QUIRÓS** THE PRICING OF SYSTEMATIC LIQUIDITY RISK IN STOCK MARKETS  
**J. L. MIRALLES QUIROS**

**MARIA ISABEL CLÍMACO / LUÍS MOURA RAMOS** QUESTIONING RATIONALITY: THE CASE FOR RISK CONSUMPTION

**HAYKEL HADJ SALEM** IMPACTES DES ACCORDS LE LIBRE ÉCHANGE EURO-TUNISIEN



## Short-term Deviations from Monetary Policy Commitment in a Monetary Union: the Degrees of Freedom of an Independent Central Bank<sup>1</sup>

Muriel Dal-Pont / Dominique Torre / Elise Tosi

GREDEG-CNRS – University of Nice Sophia Antipolis

resumo

résumé / abstract

**Este artigo apresenta um modelo que analisa o grau de autonomia de um Banco central independente, mas comprometidos com determinados objectivos, numa união monetária. Neste modelo, os resultados nominais e reais são determinados pelas interações entre os diversos agentes, as autoridades políticas supranacionais e o Banco central da união. A informação imperfeita sobre os choques que afectam a oferta, os canais de transmissão e as expectativas de médio prazo podem levar o Banco central a desviar-se dos objectivos propostos. Essa possibilidade de desvio aplica-se a bancos centrais sem condicionantes inflacionárias e empenhados exclusivamente em cumprir um objectivo nominal. Mostra-se que, em determinadas condições, os desvios nominais relativamente aos objectivos propostos não são observáveis pelos agentes nem pela autoridade supranacional que periodicamente selecciona a constituição do Conselho para a Política Monetária do Banco central. Esses desvios aumentam a variação dos valores nominais, mas diminuem as flutuações do rendimento real. Os resultados confirmam obtidos confirmam a posição defendida por Cukierman e Metzler sobre a eficiência de um comportamento ambíguo de um Banco central, neste caso numa união monetária.**

Cet article présente un modèle qui analyse le degré d'autonomie, dans une union monétaire, d'une Banque centrale indépendante mais engagée. Dans ce modèle, les résultats nominaux et réels sont déterminés par les interactions des différents agents, les autorités politiques supranationales et la Banque centrale. L'imperfection de l'information sur les chocs qui affectent l'offre, les voies de transmission et les anticipations à court terme peuvent mener la Banque à s'écarter des objectifs annoncés. C'est

surtout les banques centrales sans contraintes inflationnistes et exclusivement engagées dans la réalisation d'un objectif nominal qui sont susceptibles de présenter cet écart. On montre que dans certaines conditions les écarts nominaux par rapport aux objectifs proposés ne sont pas observables par les agents ni par l'autorité supranationale qui périodiquement sélectionne la composition du Conseil pour la politique monétaire de la Banque centrale. Ces écarts augmentent la variation des valeurs nominales, mais diminuent les fluctuations du revenu réel. Les résultats confirment la position défendue par Cukierman et Metzler sur l'efficacité d'un comportement ambigu d'une banque centrale, dans ce cas dans une union monétaire.

The paper presents a model analyzing the degrees of freedom of an independent but committed Central Bank within a monetary union. In the model, interactions between Agents, Supranational Political Authorities and the Central Bank of the Union determine the current nominal and real outcomes. Imperfectly distributed information on shocks affecting supply, transmission channels and short-term expectations create opportunities for a Central Bank to deviate from its announced objective. This opportunity to deviate especially applies to Central Banks free from any kind of inflationary bias and committed to a strictly nominal target. Under certain conditions we show that nominal deviations from stated targets are not observable either by Agents or the Supranational Political Authority that periodically selects the membership of the Council of Monetary Policy of the Bank. Those deviations increase the variance of nominal values but dampen fluctuations of real income. Our results confirm, within a monetary union, the position defended by Cukierman and Metzler concerning the efficiency of a Central Bank's ambiguous behaviour.

<sup>1</sup> The authors are grateful to O. Bruno, E. Dharmon, R. Dutu, E. Farvaque, J.F. Goux, J. Hellier, D. Plihon, M.-A. Prunier, C. Bayoud and E. Saltari and an anonymous referee of *Notas Económicas* for their helpful comments on a preliminary version of this paper. The usual disclaimers apply.

## 1. Introdução



This paper aims to contribute towards the discussion on the degrees of freedom that an independent and committed Central Bank may benefit from. These *margins of manoeuvre* consist of the Bank's residual opportunities for action. They correspond to a category of discretionary actions that it can undertake which are not really discernable to Agents and hence do not run the risk of the Council of Monetary Policy being sanctioned. Nevertheless, the actions result from the voluntary behaviour of the issuing institution when it decides that under certain circumstances it becomes preferable not to rigidly respect its commitment, either in its own interests or in the interests of the Agents or one of the political authorities. We examine the conditions under which these initiatives emerge. We debate whether they are suited to a process in which the governors of an independent Bank are appointed by the representatives of the nations who form the monetary union. Finally, we evaluate the benefits of such margins of autonomy, from the point of view of the evolution of nominal and real values over time, this being a way to deal with their potential macroeconomic efficiency.

The remainder of this paper is organized as follows: *section 2* defines the objective of the paper in relation to the main results of the literature devoted to independence and commitment. *Section 3* proposes a model of independent Central Bank behaviour within the framework of a monetary union. *Section 4* determines the current level of inflation and output in this context and studies the degrees of freedom the Central Bank has when shocks affect fundamentals and monetary channels. It presents the opportunity for committed Central Banks to deviate from their declared monetary target when some uncertainty is attached to monetary transmission channels. The result illustrates Cukierman-Metzler's (1986) conclusions, recently discussed by Faust and Svensson (2000a and b). It is interesting to note that this incentive to deviate occurs particularly successfully when Central Banks are committed to the long-term inflationary target of the economy (free from inflationary bias).

*Section 5* deals with the selection of the Council of Monetary Policy by the Supranational Political Authority and ends with the existence of a simple relationship between the respective levels of conservatism of the Supranational Political Authority and of the Central Bank. *Section 6* examines the macroeconomic impact of cases in which the Central Bank deviates from its target. Short-term deviations from monetary targets result in an increase in the variance of nominal values and in the dampening of real income fluctuations. *Section 7* concludes the analysis.

## 2. Central Bank independence and commitment in the making of monetary policy

Central Banks play an essential role in short-term economic stabilization. The stability of nominal values being a prerequisite for the development of long-term projects, these banks are in charge of accompanying growth by liberating fiscal and structural policies from nominal contingencies. Price stability is supposed to be best achieved by the independence of Central Banks. "The foundation of this assumption rests on the context of intertemporal games between the government and Agents, the delegation of monetary policy to an external agency more committed than Agents to price control being considered the optimal means of avoiding the time-inconsistency of announced inflationary policies" (Bagella and Becchetti, 1998). The Bank's mandate can be understood in terms of a *principal-agent* approach (M. J. Fratianni, J. von Hagen and C. Waller, 1997).

When Central Banks are primarily interested in achieving nominal targets they tend to be much more credible than in any other context. Since Barro – Gordon's works, it has been admitted that independent Central Banks are the ones most able to adopt a form of behaviour they cannot deviate from, thus ensuring their *credibility*. Exploiting and justifying their independence, the Central Banks are ready to engage in the implementation of a permanent fight against inflation.



The commitment of Central Banks to long-term or medium-term objectives has been considered for some time as "superior to discretion, which can involve a stabilization bias" (Evans and Honkapohja, 2002, p. 2). As in the case of the ECB, commitment to a strict monetary goal explicitly defined by the Bank would avoid any inflation bias or time inconsistency which would, in turn, lead to immediate and very serious sanctions on the part of the Agents. Making a commitment and respecting it have therefore been considered two essential aspects of a Central Banker's attitude whenever the Central Bank pursues a nominal objective.

This point of view, which stems from widely acknowledged theoretical results, is being increasingly challenged by literature as well as Central Bank practices. An independent Central Bank deviating temporarily, and without revealing this, from its initial commitment could benefit from greater flexibility than a government directly administering a monetary policy. This thesis seems to be admitted by the ECB itself: "the solution of an independent Central Bank allows a greater flexibility than the adoption of a strictly mechanical rule" (Issing, 2000). A. Muscatelli and C. Trecroci (2000) also deal with monetary policy rules and show that the literature generally overestimates the importance of independence and assumptions of commitment in the selection of a rule aiming at fighting inflation.

One way – amongst others – to challenge the constant meeting of Bank commitments is the discussion on the respective benefits of *transparency* and *ambiguity* in relation to Central Banks. A Central bank is transparent if it can be easily verified that it tries to achieve what it declares to be its goal. The *transparency* of Central Bank actions is often considered a guarantee of its credibility. When credibility is subordinated to transparency, the Central Bank must not only respect its final objectives but must also reveal its intermediary objectives, therefore sacrificing any short-term appreciation of the opportunity for monetary intervention. By casting a certain level of *ambiguity* over its short-term reactions, it can, on the other hand, exploit degrees of freedom and adapt its intervention to certain phases of economic activity (Cukierman and Meltzer, 1986). The theoretical literature devoted to the determination of optimal behaviour in the field of monetary policy reviews these notions of transparency, ambiguity and inertia levels applied to Central Bank behaviour (Faust et Svensson, 1999, 2000). The opacity of Central Bank behaviour may result from "the absence of clear knowledge about the 'true' model of the economy and sometimes is due to the attempt of policymakers to hedge their positions in the face of model and of political uncertainties" (Cukierman, 2002, b, p. 16). Recent advances in literature devoted to transparency and optimality in monetary policy do not provide a definitive answer in terms of the social desirability of total transparency (Cukierman, 2001, Jensen, 2001, Geraats, 2003).

Without direct connection to this strand of analysis, S.C.W. Eijffinger and M. Hoeberichts (1998) have argued that independence and conservatism are no longer joint attributes of the Central Banks, but substitutes. With greater independence, the Bank would be exempt from adopting an attitude as conservative as that of an issuing institution depending on the government. The Bank would be allowed deviations from strictly conservative principles (the achievement of a pure monetary target) if its decisions resulted from its appreciation of the current state of the economy and from its disinterested perception of monetary or real degrees of flexibility (see Berger, De Haan and Eijffinger, 2001). Whilst opportunities for deviation may be economically desirable (see Denicolò, 1998), committed and transparent Central Banks cannot support them. At this stage of the analysis, the choice seems to be between a committed, conservative, transparent and inflation-bias free Central Bank and a non-credible, accommodative and temporally inconsistent one.

The following sections attempt to explore the conditions under which it would be possible to escape, at least temporarily, from this dilemma, so that Central Banks are independent, subject to commitment inside a monetary union, openly conservative, but tacitly attempting to dampen the short-term real shocks. The degree of freedom an independent Central Bank possesses is then the joint consequence of imperfectly distributed information on the transmission channels of monetary policy and the adequate counter-cyclical short-term actions of the monetary authorities.

### 3. The model



The economy is composed of several identically-sized countries forming a monetary union. The current decisions of the Agents living in this area contribute towards determining the global income. Every country periodically chooses its political personnel. National elections are uniformly distributed through time and the voting figures are identical across member-states. A Supranational Political Authority composed of representatives of the different national governments periodically appoints the Monetary Policy Council. This council is actually the decision-making body of the union's Central Bank; its mandate is more extensive than that of the national governments. The choice of the monetary authority is made on the basis of its ability to accomplish the long-term objectives of the Political Authority. As a counterpart to the Bank's independence which avoids any time inconsistency in monetary policy, the Bank is required to be committed to reaching an inflation target. Non-compliance with this commitment is sanctioned by non-renewal of the mandate of the members of the Monetary Policy Council. This Council has its own objectives which it seeks to achieve within the constraint that its actions must allow the mandate of its members to be renewed.

#### 3.1. Agents

Agents have a nominal reference  $\bar{\pi}$  known to the political and monetary authorities and representing the desirable long-term inflation rate. In the short term, this reference is corrected by a random term  $\varepsilon_t$  whose expected value is zero and whose variance is constant  $\sigma_\varepsilon^2$ , revealing a shock on preferences that is not observed by the monetary authorities:

$$\Pi_t^* = \bar{\pi} + \varepsilon_t \quad (1)$$

With this expression, we assume that the inflation reference of Agents is permanently but stationarily adjusted according to the macroeconomic context and outcomes.

During each period, the Agents determine the real income  $Y_t$  of the area, as a result of their decisions concerning production and consumption. This income has a structural component  $Y^N$ , which could be interpreted as its "natural" level. The gap between  $Y_t$  and  $Y^N$  depends on three terms. The first is a linear function of the monetary surprise  $[\Pi_t - \bar{\pi}_t]$ , where  $\Pi_t$  represents the effective level of inflation and  $\bar{\pi}_t$  is the short-term expectation previously formulated by Agents. This term corresponds to the transitory monetary illusion generated by non-expected inflation. The second term integrates the discrepancy  $[\Pi_t^* - \Pi_t]$  between the short-term inflation reference and the effective inflation rate. It takes into account the level of the Agents' 'aversion to inflation' and could be considered a static proxy of the cost expressed in terms of output generated by time-inconsistent expected monetary policies. Finally, a non-observed supply shock takes the form of the random variable  $u_t$  whose expected value is zero and which has a constant variance  $\sigma_u^2$ .

$$Y_t = Y^N + \lambda[\Pi_t - \bar{\pi}_t] + \mu[\Pi_t^* - \Pi_t] + u_t \quad (2)$$

with  $\lambda > 0$  and  $\mu > 0$ .

#### 3.2. The Supranational Political Authority

The long-term inflationary reference of the Supranational Political Authority is the rate  $\bar{\pi}$ , that is, the long-term Agents' reference. Its short-term reference  $\Pi_{st}^*$  is based on this same rate  $\bar{\pi}$  permanently corrected by an inflation bias  $A$  and by a random term  $v_t$  whose expected value is



zero and whose variance is  $\sigma_v^2$ . The inflation bias introduces an element of time-inconsistency into the political choices; it comes from the pre-electoral context in which a fraction of the members of the political authority are to be found at any given time.

The random term accounts for the impact of exogenous economic or political factors affecting the union in the short term.

$$\Pi_{st}^* = \bar{\Pi} + A + v_t \quad (3)$$

The supranational authority has mixed objectives that can take the form of a loss function whose expected value is minimized whenever a new Monetary Policy Council is chosen. This function is expressed as follows at any period:

$$L_{st} = \frac{1}{2} \left[ (Y_t - Y^N)^2 + \alpha' \beta' (\Pi_t - \Pi_{st}^*)^2 + \alpha' (1 - \beta') (\Pi_t - \bar{\Pi})^2 \right] \quad (4)$$

Its quadratic form is related to the increasing sensitivity of the political authority in relation to the discrepancies between current and reference values. Its real term is standard. Its nominal term is double: on one hand the political authority is sensitive to the difference between current inflation  $\Pi_t$  and its short-term reference  $\Pi_{st}^*$ ; on the other hand it is also sensitive to the gap between current inflation and the long-term reference  $\bar{\Pi}$ . The first of both nominal terms results from the aggregation of the individual behaviour of the union's members: it accounts for the renewal objectives of each national political authority, usually inducing an inflationary bias. The second term takes into account the non-renewal threat through the future election of dissenting political authorities, should inflation diverge too far from the long-term Agent reference. Non-individualistic behaviour or the search for a common interest by the members of political authority may be relevant attitudes in relation to avoiding individual sanctions. It is an economic counterpart of the fact that the national political parties tend to be considered the accountants of the union's performance as soon as they take on responsibilities and agree to remain within the union, regardless of any reservations they have previously expressed – or they would express in opposition. The coefficient  $\alpha'$  becomes higher as the share of the nominal target gains in importance; the coefficients  $\beta'$  and  $(1 - \beta')$ , where  $\beta'$  belongs to the interval  $[0, 1]$  determine the relative weight of the two components of nominal values.

The Supranational Political Authority is supposed to have no means of controlling inflation and income in the short term except by contributing to non-expected shocks in preferences and supply. Its unique role is to periodically appoint the Monetary Policy Council of the Central Bank, which is in charge of monetary policy.

### 3.3 The Central Bank of the Monetary Union

The Central Bank uses different monetary and non-monetary tools (adjustment of key interest rates, management of minimum reserves, open market interventions, announcements and comments on the current situation...) to carry out its monetary policy. Each of these tools has effects that are not fully predictable due to the complexity of the monetary transmission channels and the variability of opinions. The Central Bank has a short-term inflation reference  $\Pi_{cbt}^*$  communicated at the moment the (new) Monetary Policy Council is officially placed in charge of monetary policy. It also takes into account the long-term reference of the economy (that is the expected value of the short-term reference of the Agents which is not observed by the Bank) and the short-term reference of the Supranational Political Authority, the Bank being unable to observe the random component  $v_t$ . We assume that the Central Bank corrects the long-term reference of the economy,  $\bar{\Pi}$  by a fraction of the expected difference between the Agent and the political authorities' reference:

$$\Pi_{cbt}^* = \bar{\Pi} + \theta (E_t(\Pi_{st}^*) - \bar{\Pi}) \quad (5)$$

The particular case  $\theta = 0$  corresponds to a Central Bank announcing a constant inflation rate targeting (like the early ECB).

The case  $\theta = 1$  corresponds to a Central Bank smoothing the political authorities' target. It will be verified that the inflationary bias is then fully transmitted: this would be the sign of a weak of effective independence on the part of the monetary authorities.

In the intermediate case, the Central Bank transmits only a fraction of the bias.

Taking into account its observational abilities and its expertise, the Central Bank observes the supply shock  $u_t$  and it chooses  $\Pi_{cbt}$ , the inflation level for the period. However, the Bank does not control the transmission channels of monetary policy perfectly because of the existence of endogenous components in the money creation process (cf. Dal-Pont, Torre and Tosi, 2000) or other kinds of uncertainty regarding the determination of the price level (see Geraats, 2001, Gerdesmeier, Motto and Pill, 2002). The effective inflation is then equal to the level targeted by the Bank  $\Pi_{cbt}$  incremented by a random term  $\delta_t$  whose expected value is zero and whose variance is  $\sigma_\delta^2$ .

$$\Pi_t = \Pi_{cbt} + \delta_t \quad (6)$$

The Central Bank does not observe  $\delta_t$  but knows its distribution. Agents and political authorities know of the existence of imperfections relative to the transmission channels; they know that the expected value of  $\delta_t$  is zero, but they cannot observe  $\sigma_\delta^2$  as they do not observe  $\Pi_{cbt}$ . Consequently, the Bank has the opportunity to periodically target an inflation level  $\Pi_{cbt}$  that is different from  $\Pi_{cbt}^*$  without revealing its transitory deviation from its long-term target. In relation to this targeted inflation rate, the Bank can then anticipate an effective inflation level given by the expected value of  $\Pi_t$  that is  $\Pi_{cbt}$ . It deduces from this an expectation  $E_{cbt}(Y_t)$  of the current level of income that in particular takes into account its own observation of the level of inflation expected by the Agents, the expected value of their short-term inflationary reference and the Bank's observation of the supply shock:

$$E_{cbt}(Y_t) = Y^N + \lambda[\Pi_{cbt} - \bar{\Pi}_t] + \mu[E(\Pi_t^*) - \Pi_{cbt}] + u_t$$

hence,

$$E_{cbt}(Y_t) = Y^N + \lambda[\Pi_{cbt} - \bar{\Pi}_t] + \mu[\bar{\Pi} - \Pi_{cbt}] + u_t \quad (7)$$

Like the political authorities, the Central Bank has a quadratic loss function given as (8):

$$L_{st} = \frac{1}{2} \left[ (E_{cbt}(Y_t) - Y^M)^2 + \alpha(\Pi_{cbt} - \Pi_{cbt}^*)^2 \right] \quad (8)$$

This loss function includes a real term, meaning that the monetary authorities are not only concerned about stabilizing inflation but also dampening the fluctuations of real output. This real component of Central Bank behaviour accounts implicitly for intertemporal smoothing gains for the Agents. We do not introduce any explicit welfare relation which would describe this





preference of agents for a certain level of output stability with precision. Even if it exists, the function (stable or not) describing this kind of preference is not observed by the Central Bank which restricts itself to considering stabilizing output as one aspect of its duty. More precisely, this function considers the gap between the level of the expected income  $E_{cbl}(Y_t)$  and its natural level  $Y^N$ . The Bank has a single nominal indicator: it is sensitive to the discrepancy between its short-term effective and announced targets  $\Pi_{cbl}$  and  $\Pi_{cbl}^*$ . The coefficient  $\alpha$  ( $\alpha > 0$ ) measures the degree of conservatism of the monetary authority.

The interpretation of this function is as follows: for the authority, deviating from the announced target may give rise to sanctions but it can do so if the aim is the reduction of a discrepancy that is too large in relation to the real target. Agents do not know the form of the loss function of the Bank, given that the Bank is committed to respecting a nominal target. Under appropriate conditions, Agents would impute the transitory deviations from nominal announced target to imperfections in transmission channels. When  $\Pi_{cbl}$  and  $\Pi_{cbl}^*$  are distinct, the difference is termed  $\gamma_t$ :

$$\gamma_t = \Pi_{cbl} - \Pi_{cbl}^* \quad (9)$$

Agents do not observe the deviation of the targeted inflation from its announced level but only see the global difference between the effective rate of inflation and the announced target (10):

$$\Pi_t - \Pi_{cbl}^* = \delta_t + \gamma_t \quad (10)$$

#### 4. Prices and Income in the short-term

We introduce normalized notation, then present the sequence of short-term events. Finally, we determine the real and nominal outcomes for the short-term period.

##### 4.1. Normalized variables and the reduced form of the Model

For the sake of simplicity, we define variables in normalized values using long-term references of inflation and income as normalized references. These new variables can be restated as follows:

- *Supranational Political Authority*

$$\pi_{st}^* = \Pi_{st}^* - \bar{\Pi}, \text{ as the short-term reference for the Supranational Political Authority}$$

- *Agents*

$$\pi_t^* = \Pi_t^* - \bar{\Pi}, \text{ as the short-term inflation reference for Agents}$$

$$\tilde{\pi}_t = \bar{\Pi}_t - \bar{\Pi}, \text{ as the short-term inflation level expected by Agents}$$

- *Central Bank*

$$\pi_{cbl}^* = \Pi_{cbl}^* - \bar{\Pi}, \text{ as the official (subject to commitment) level of inflation for Central Bank}$$

$$\bar{y}_{cbl} = E(Y_{cbl}) - Y^N, \text{ as the real current income expected by Central Bank}$$

$$\pi_{cbl} = \Pi_{cbl} - \bar{\Pi}, \text{ as the effective short-term level of inflation targeted by Central Bank}$$

- *Short term outcomes for the economy*

$$\pi_t = \Pi_t - \bar{\Pi}, \text{ as the current level of inflation}$$





$y_t = Y_t - Y^N$ , as the level of current real income

• *The Model in its reduced form*

In reduced form, using normalized variables, the system (1) – (10) is summed up in equations (11) to (16):

$$y_t = \lambda[\pi_t - \bar{\pi}_t] + \mu[\epsilon_t - \pi_t] + u_t \quad (11)$$

$$L_{cbt} = \frac{1}{2} \left[ (\bar{y}_{cbt})^2 + \alpha(\pi_{cbt} - \theta A)^2 \right] \quad (12)$$

$$\bar{y}_{cbt} = \lambda[\pi_{cbt} - \bar{\pi}_t] - \mu\pi_{cbt} + u_t \quad (13)$$

$$\gamma_t = \pi_{cbt} - \theta A \quad (14)$$

$$\pi_t = \pi_{cbt} + \delta_t \quad (15)$$

$$L_t = \frac{1}{2} \left[ (y_t)^2 + \alpha' \beta' (\pi_t - A - \nu_t)^2 + \alpha' (1 - \beta') (\pi_t)^2 \right] \quad (16)$$

#### 4.2. The sequence of short-term events

In the short term, the sequence of events is as follows:

1. Agents determine their short-term expectations for inflation  $\bar{\pi}_t$  which becomes public information while their short-term inflationary reference (their opinion about what would currently be the "normal rate of inflation") depends on non-revealed preference shocks  $\epsilon_t$ .
2. The Central Bank observes the Agents' Inflationary Expectations (publicly recorded in sample surveys) and the supply shock  $u_t$  but not the periodic shock on Agent preferences  $\epsilon_t$ . Through minimization of (12), the Central Bank determines the current rate of inflation  $\pi_{cbt}$  that it aims to reach using (13) to account for real retroactions. This rate of inflation may differ from the announced target  $\pi_{sr}^*$ . From (14) the deviation  $\gamma_t$  can be deduced from the effective nominal target to the target subject to Central Bank commitment.
3. The impact of the shock  $\delta_t$  on the transmission channels determines the current rate of inflation  $\pi_t$  according to (15). The current income  $y_t$  is obtained from (11) by the integration of the impact of the shocks on preferences  $\epsilon_t$ . Because Supranational Political Authorities have no influence on monetary policy in the short term, the "macroeconomic shock"  $\nu_t$  and the loss function (16) do not interfere with the determination of the current nominal and real values.

#### 4.3. Short-term Outcomes and Discussion

As a result, we can evaluate  $\pi_t$  the deviation between the current level of inflation and the long-term inflationary reference and  $y_t$ , the deviation of the current income from its natural level:

$$\pi_t = m\alpha\theta A + m(\lambda - \mu)\lambda\bar{\pi}_t - m(\lambda - \mu)u_t + \delta_t \quad (17)$$

$$y_t = m(\lambda - \mu)\alpha\theta A - m\alpha\lambda\bar{\pi}_t + m\alpha u_t + (\lambda - \mu)\delta_t + \mu\epsilon_t \quad (18)$$



$$\text{with } m = \frac{1}{(\lambda - \mu)^2 + \alpha} > 0$$

It can be observed that the difference between the effective inflation rate and the Agent long-term reference depends on four factors: the inflationary bias of the political authorities, Agent expectations, supply-side shocks and imperfections in the transmission channels of monetary policy. The inflationary bias and the failures of the transmission mechanisms act in the expected direction. However, the impact of Agent expectations and the shock on the supply-side depend on the relative weight of the “monetary surprise” and the “aversion to inflation” effect.

The more Agent behaviour depends on the monetary illusion phenomena, the more inflation expectations will be self-fulfilling and the supply shocks will be deflationary. The inverse relationship can be observed when aversion to inflation dominates the monetary illusion phenomena.

The gap between short-term income and its natural level depends on the same factors but also on preference shocks<sup>2</sup>. The causality between income variations and their determinants is logical with intuition. We once again confirm the influence of the relative weight of the two components of Agent reaction to prices in determining the sign of the relations between the inflationary bias and income on the one hand, and between shocks on the propagation channels and income on the other hand.

The observed level of inflation comes from the adoption by the Central Bank of an effective target  $\pi_{cbt}^*$  generally differing from the announced target  $\pi_{cbr}^*$ . Then, we obtain:

$$\pi_{cbr} = m\alpha\theta A + m(\lambda - \mu)\lambda\bar{\pi}_t - m(\lambda - \mu)u_t \quad (19)$$

that is a target differing from  $\pi_{cbr}^*$  with,

$$\pi_{cbr}^* = \theta A$$

As a special case, one can determine the nominal and real magnitudes associated with rational expectations. Instead of  $\bar{\pi}_t$ , we therefore consider the expected value of the short-term rate of inflation, given the observation associated with the action of the same components of the Monetary Policy Council. This rationally expected inflation rate  $\pi_t^*$  results from the following expression:

$$\pi_t^* = E(m\alpha\theta A + m(\lambda - \mu)\lambda\pi_t^* - m(\lambda - \mu)u_t + \delta_t)$$

that is,

$$\pi_t^* = \frac{m\alpha\theta A}{1 - m(\lambda - \mu)\lambda}$$

In this case, the current nominal and real magnitudes depend only on the inflationary bias and on the shocks on supply, preferences and monetary transmission channels. From (17), one can for instance, deduce the current rate of inflation when expectations are rational:

<sup>2</sup> A positive preference shock is associated with a weaker Agent reaction sanctioning an inflation gap (cf. equation 7). This positive shock then results temporarily in a higher level of real output.



$$\pi_t = \frac{m\alpha\theta A}{1 - m(\lambda - \mu)\lambda} - m(\lambda - \mu)u_t + \delta_t$$

This particular case, as well as the general one given by equations (17) and (18), does not take into account the incentive constraint faced by the Central Bank. Indeed, the Bank cannot choose a target that is different to the one announced, except when the Political Authorities are not able to disclose such a deviation. In the short term, deviations cannot be observed because of the existence of transmission channels and endogenous money, which are both responsible for the imperfect control of inflation. If the long-term average level of inflation is different from its announced target, it must be concluded that the short-term deviations are not only the consequence of zero mean error terms. In this case, it can be deduced that the monetary authority targets at least periodically an unannounced level of inflation. However, the variance of the inflation rate does not provide any information, since Agents do not know it. From equation (17), this long-term average level of inflation can be expressed, which could be considered an adequate way to estimate the nature of the targeted rate of inflation by the monetary authorities since, on average, the shocks on monetary transmission vanish.

$$E(\pi_t) = m\alpha\theta A + m(\lambda - \mu)\lambda E(\tilde{\pi}_t) \quad (20)$$

If we assume that the long-term expected rate of inflation is equal to the mean of the expected short-term rates of inflation, the expression reduces to:

$$E(\pi_t) = \frac{m\alpha\theta A}{1 - m(\lambda - \mu)\lambda}$$

This expression<sup>3</sup> differs from  $\pi_{cbt}^*$  which reduces to  $\theta A$ . It may be concluded that when the monetary authority transmits a part of the inflationary bias, the Central Bank cannot deviate from its announced target without these deviations being revealed in the long run. Given the value of  $m$ , it is, however, possible to determine specific cases of identity between  $E(\pi_t)$  and  $\pi_{cbt}^*$ . Conditions (i), (ii) and (iii) define three sufficient conditions for the cancellation of any gap between  $E(\pi_t)$  and  $\pi_{cbt}^*$ :

$$(i) \quad \lambda = \mu, \forall \theta$$

$$(ii) \quad \mu = 0, \forall \theta$$

$$(iii) \quad \theta = 0, \forall \alpha, \lambda, \mu$$

In case (i), Agent actions are independent of the rate of inflation; in this extreme situation, monetary policy has no impact in real terms and monetary policy has no reason to exist anymore.

The second case (ii) corresponds to an absence of inflation aversion on the part of Agents (or to the absence of any sanction in real terms for an inflationary policy); in this case, given that the shocks are of zero mean, the only role of the Bank is to accommodate the inflation expectations

<sup>3</sup> It should be noted that the expected long-term rate of inflation is in this case the rationally expected short-term rate.



of Agents and supply shocks, both having been previously observed. In real terms no sanctions will be incurred.

The third case (iii) is more interesting (and maybe more realistic). It represents a situation in which the monetary authority is committed to a target assimilated within the long-term reference of the economy. In this case, the Central Bank does not transmit the inflationary bias of the political authorities. With a strict short-term commitment to the target, the long-term inflation reference of Agents thus permits the Central Bank to introduce short-term transitory deviations from its nominal objective. In this case, the deviation from the target is compatible with the minimization of (13). If the autonomy of a Central Bank is defined by its capacity to deviate from the stated objectives, it can, paradoxically, be autonomous whilst committed to a target corresponding to the long-term reference of the economy and incidentally insensitive to the inflationary bias of the monetary authorities.

## 5. The appointment of the Council of Monetary Policy

The appointment of the Council of Monetary Policy is the outcome of a process in which several teams apply for the forthcoming mandate. Since the political authority does not know the loss function of the candidates, the only way to disclose the future behaviour of the candidates is by asking each team to reveal the expected distribution (mean-value and variance) of the rates of inflation that would result from its own actions<sup>4</sup>.

### 5.1. The Submission of Candidates' Applications

For the sake of simplicity, we will assume that all the shocks on expectations, preferences, supply, inflationary bias and transmission channels are independently distributed. We will restrict the analysis to case (iii) defined above, assuming that the applications are limited to candidates whose short-term inflation target is the long-term reference of the economy. Candidate teams are thus only differentiated by their respective levels of conservatism  $\alpha$ , which are not directly observed by the Political Authority. Each of them uses the publicly known exogenous distributions of  $\tilde{\pi}_t, u_t, \delta_t, \epsilon_t$ , to inform the Political Authority of their own distributions of nominal and real magnitudes  $\pi_t$  and  $y_t$ . In doing so, they implicitly solve our *section 4* problem and express  $\pi_t$  and  $y_t$  as functions of their respective level of conservatism  $\alpha$ , according to (21) and (22):

$$\pi_t = m\lambda\tilde{\pi}_t - mu_t + \delta_t \quad (21)$$

$$y_t = -\eta\lambda\tilde{\pi}_t + \eta u_t + \mu\epsilon_t + (\gamma - \mu)\delta_t \quad (22)$$

with,

$$m = \frac{(\lambda - \mu)\lambda}{(\lambda - \mu)^2 + \alpha} \quad \text{and} \quad \eta = \frac{\alpha}{(\lambda - \mu)^2 + \alpha}$$

### 5.2. The Appointment of the Board

The Political Authority is only informed of the distribution of  $\pi_t$  and  $y_t$  for every applicant team. These magnitudes include the periodic loss function of the Authority whose expected value is minimized, as the choice of Central Banker has long-term consequences:

<sup>4</sup> We could also imagine that a numerical simulation or a deeper technical discussion between the teams present and the political authority could enable it to apprehend the distribution of price evolution associated with each team.

$$E(L_t) = \frac{1}{2} \left[ E \left( (y_t)^2 \right) + \alpha' \beta' E \left( (\pi_t - \pi_{st}^*)^2 \right) + \alpha' (1 - \beta') E \left( (\pi_t)^2 \right) \right]$$

Hence,

$$\begin{aligned} E(L_t) = & \frac{1}{2} \left[ \text{Var} (y_t) + \left( E (y_t) \right)^2 + \alpha' \beta' \text{Var} (\pi_t - \pi_{st}^*) \right] \\ & + \frac{1}{2} \left[ \alpha' \beta' \left( E(\pi_t - \pi_{st}^*) \right)^2 + \alpha' (1 - \beta') \text{Var} (\pi_t) + \alpha' (1 - \beta') \left( E(\pi_t) \right)^2 \right] \end{aligned} \quad (23)$$

The Political Authority knows the distribution of the shocks on the inflation bias  $v_t$ , which is a component of  $\text{Var} (\pi_t - \pi_{st}^*)$  and of  $E(\pi_t - \pi_{st}^*)$ . From each distribution of prices and the corresponding distribution of income calculated in (12), the Political Authority is thus able to infer the expected value of its loss-function. The political authority uses this whole set of information to minimize the expected value of (16) explicitly as a function of the distribution of prices corresponding to each candidate team and implicitly as a function of the level of conservatism  $\alpha$  of these teams. Consequently, it chooses the team best suited to form the Council of Monetary Policy by implicitly selecting the level of conservatism and minimizing its loss function.

In the case where there is a continuum of candidate teams, each being defined according to their respective level of conservatism, the selected value of  $\alpha$  will correspond to the minimization of (23), i.e. in the case of non-correlated shocks (see annex for the details of calculations),

$$\alpha^* = \lambda^2 \alpha'$$

In this case, the relationship defines non-correlated shocks and the relation of proportionality between the respective levels of conservatism of the Political Authority and of the selected team comes from intuition (see annex 1 for a generalization of the relation between  $\alpha$  and  $\alpha'$ ). Note that whatever the level of correlation between shocks, restricting the applications to candidate teams free from any inflationary bias will exclude all the influence of the Political Authority's inflationary bias  $A$  on the optimal level of conservatism of the selected team. Secondly, whatever their respective level of conservatism, the teams applying have no ability to stabilise the consequences of nominal target variations on prices and income, since the shocks on preferences are only observable *ex-post*. This explains why  $\alpha^*$  does not depend on  $\mu$ . Thirdly, given  $\pi_{st}^*$  and the nominal part of (23), the higher  $\alpha'$  is, the higher  $\alpha^*$  would be, as the optimal degree of conservatism of the Central Bank from the point of view of the Governments. Finally, the level of proportionality from  $\alpha'$  to  $\alpha^*$  depends on the need to control the effect on observed shocks, i.e. the shocks on expectations<sup>5</sup> and on supply, which explains the role of  $\lambda$  in the proportionality relationship between the two degrees of conservatism.

## 6. Commitment and Degrees of Freedom for Monetary Stabilization

The *margins of manœuvre* of the monetary authorities in the short term and the fact that they do not feel absolutely committed providing they deviate within a range of values that is not observable, are natural consequences of the existence of a loss function including real terms if the Central Bank's actions are not perfectly observable. Such behaviour will be socially beneficial if, and only if, gains can be identified on a macroeconomic level. From this perspective, a new

<sup>5</sup> When expectations are rational, this first term naturally vanishes and the supply shock remains the only short-term perturbation under the Bank's control.





loss function in relation to Agents could be introduced. This function would compare the distribution of nominal and real values associated both with a rigid respect for commitment and with the deviation scenario linked to the imperfection of monetary transmission channels. Alternatively and less precisely, it is possible to compare the descriptors of inflation and income related to the different possible Central Bank actions.

If the Central Bank remains committed, the inflation rate effectively targeted is given by  $\pi_{st}^*$ . This rate is corrected by the monetary policy shock in order to determine the observed inflation rate (24).

$$\pi_t = \theta A + \delta_t \quad (24)$$

Then, the income of the period comes from (25):

$$y_t = (\lambda - \mu)\theta A + (\lambda - \mu)\delta_t - \lambda \bar{\pi}_t + \mu \epsilon_t + u_t \quad (25)$$

Monetary policy introduces a long-term inflationary bias, detected by the fact that the expected value of the long-term inflation level is not equal to the long-term reference of the economy. Only case (iii) analyzed in section 5 excludes the existence of such a bias: when the Central Bank is not sensitive to the political authorities' bias, (that is  $\theta = 0$ , this case being assimilated to the European Central Bank's target of 2%), which is the case under analysis. When the Central Bank effectively does not deviate in the short term, we then obtain:

$$\pi_t = \delta_t \quad (26)$$

$$y_t = -\lambda \bar{\pi}_t + u_t + \mu \epsilon_t + (\lambda - \mu)\delta_t \quad (27)$$

It can be observed that here the expected values of  $\pi_t$  and of  $y_t$  are actually equal to zero. Variances are given by the following expressions:

$$Var(\pi_t) = \sigma_\delta^2 \quad (28)$$

$$Var(y_t) = \lambda^2 Var(\bar{\pi}_t) + \sigma_u^2 + \mu^2 \sigma_\epsilon^2 + (\lambda - \mu)^2 \sigma_\delta^2 \quad (29)$$

Both variances should be compared to the ones that come from the short-term degree of Central Bank freedom given by equations (21) and (22) in the case where  $\theta = 0$ . We then obtain:

$$Var(\pi_t) = m^2 \lambda^2 Var(\bar{\pi}_t) + m^2 \sigma_u^2 + \sigma_\delta^2 \quad (30)$$

$$Var(y_t) = \eta^2 \lambda^2 Var(\bar{\pi}_t) + \eta^2 \sigma_u^2 + \mu^2 \sigma_\epsilon^2 + (\lambda - \mu)^2 \sigma_\delta^2 \quad (31)$$

Referring to the value of  $\eta$ , it can easily be verified that, in general, using some remaining degrees of freedom inside its binding commitment, the Central Bank increases the variance of

inflation but decreases the variance of income. Compared to a central institution which rigorously respects its commitment, a Central Bank accurately deviating in the short term will use monetary policy to partially dampen both the supply shocks and the impact of short term expectations. At the same time it will be able to improve output stabilization.



## 7. Concluding Comments

The aim of this paper is to examine the degrees of freedom of a Central Bank whose Council of Monetary Policy is periodically appointed by a Supranational Political Authority within the framework of a monetary union. This Council has to be committed from the moment its members are renewed and this commitment is verifiable through the results of the Bank's actions. However, the existence of imperfections that are not fully observable, associated with monetary policy channels allows the Bank to deviate periodically from its target without revealing that it does not effectively respect its commitment. The result obtained by A. Cukierman and A. H. Meltzer (1986) and recently discussed by J. Faust et L. E. O. Svensson (2000, a and b) can be extended: in certain cases that are very significant in the context examined here, maximum transparency is not desirable in the realization of the Central Bank's commitment. It is interesting to note that the degree of freedom exploited by the monetary institution may be activated when the announced objective is invariant and assimilated into the long-term reference of the economy. The Bank has all the more the opportunity to deviate from its commitment when this matches Agents' reference and neglects the inflationary bias of the political authorities. This result simultaneously justifies the independence of Central Banks and the existence of an independent Bank commitment free from any inflation bias. This situation is most likely to result in the activation of some kind of short-term "residual discretion" in order to stabilize an economy disturbed by supply shocks and characterized by volatile short-term expectations. In the absence of any commitment, there generally exists a short-term trade-off between inflation and output variability (see for instance Taylor, 1979 or Clarida, Galí and Gertler, 1999). In the case of commitment, any nominal variability in inflation generally proves to be counter-productive in real terms. Our exercise suggests that such a trade-off could equally be restated in this later case, where the actions that Central Bank can undertake concern the residual degree of freedom offered by the existence of noisy transmission channels.



## References

- Alesina, A.; Summers, L. J. (199, «Central Bank Independence and Macroeconomic Performance: Some Comparative Evidence», *Journal of Money, Credit and Banking*, 25, 2, 151-162.
- Athey, S.; Atkeson, A.; Kehoe, P. J. (2003), «The Optimal Degree of Monetary Policy Discretion», *Federal Reserve Bank of Minneapolis, Research Staff Reports*, October.
- Bagella m.; Becchetti, L. (1998), «Inflation, Central Bank Independence, Labour and Financial Governance: Some evidence from OECD Countries», in *Institutions and Economic Organization in the Advanced Economies*, edited by M. Baldassari, L. Paganetto and E. Phelps, MacMillan Press LTD.
- Banaian, K.; Burdekin, R. C. K.; Willet, T. D. (1995), «On the Political Economy of Central Bank Independence», in K. D. Hoover et S. M. Sheffrin eds., *Monetarism and the Methodology of Economics: Essays in honour of Thomas Mayer*, Aldershot, Edward Elgar, 178-197.
- Banaian, K.; Luksetich, W. A. (2001), «Central Bank independence, economic freedom, and inflation», *Economic Inquiry*, 39, 1, 149-161.
- Begg, I; Green, D. (1998), «The political economy of the European Central Bank», in P. Arestis et M. Sawyer eds., *The Political Economy of Central Banking*, Edward Elgar Cheltenham, UK.
- Berger, H.; Haan, J. de; Eijffinger, S. (2001), «Central Bank Independence: An update of Theory and evidence», *Journal of Economic Survey*, 15, 1.
- Blinder, A. (1999), *Central Banking in Theory and Practice*, The MIT Press, Cambridge, Massachusetts.
- Brociner, A.; Chagny, O. (1996), «La Bundesbank: une orthodoxie pragmatique», *Revue de l'O.F.C.E.*, 56, janvier, 95-106.
- Cukierman, A. (1992), *Central Bank Strategy and Independence*, M.I.T. Press, Cambridge Mass.
- Cukierman, A. (1994), «Commitment through delegation, political influence and Central Bank independence», in J. De Beaufort Winjholds; S. Eijffinger and L. Hooghuin eds., *A Framework for Monetary Stability*, Kluwer Academic Publishers, Dordrecht, Boston, London.
- Cukierman, A. (2001), «Accountability, Credibility, Transparency and Stabilization Policy in the Eurosystem», in C. Wyplosz, ed., *The Impact of EMU on Europe and the Developing Countries*, Oxford: Oxford University Press, 40-75.
- Cukierman, A. (2002, a), «Does a Higher Sacrifice Ratio Mean that a Central Bank Independence is Excessive?», *Annals of Economics and Finance*, 3, 1-25.
- Cukierman, A. (2002, b), «Are Contemporary Central Banks Transparent About Economic Models and Objectives and What Difference Does It Make?», *Federal Reserve Bank of St Louis, Review*, 84, 4, July-August.
- Cukierman, A.; Meltzer, A. H. (1986), «A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information», *Econometrica*, 54, 5, 1099-1128.
- Deane, M.; Pringle, R. (1994), *The Central Banks*, London: Hamish Hamilton.
- Eijffinger, S. C. W.; Hoerichs, M. (1998), «The trade-off between Central Bank independence and conservativeness», *Oxford Economic Papers*, 50, 397-411.
- Evans, G. W.; Honkapohja, S. (2002), Monetary Policy, Expectations and Commitment, *mimeo*, May 27.



Faust, J.; Svensson, L. E. O. (2000, a), «The Equilibrium Degree of Transparency and Control in Monetary Policy», *Journal of Money, Credit and Banking*, December.

Faust, J.; Svensson, L. E. O. (2000, b), «Transparency and Credibility: Monetary Policy with Unobservable Goals», *International Economic Review*, 42, 369-397.

Fratianni, M. J.; Hagen, J. von; Waller, C. (1997), «Central Banking as a Political Principal-Agent Problem», *Economic Inquiry*, 35, 378-393.

Gerdesmeier, D.; Motto, R.; Pill, H. (2002), Paradigm Uncertainty and the role of monetary developments in monetary policy Rules, *mimeo*.

Issing, O. (2000), «Should we have faith in Central Banks», allocution at the *St. Edmund's College Millenium Year Lecture*, Cambridge, 26 October.

Lin, X. (1999), «Central-Bank Independence, Economic Behaviour, and Optimal Term Lengths: Comment», *American Economic Review*, 89, 4, 1056-1062.

Lippi, F. (1998), «On Central Bank Independence and the Stability of Policy Targets», *Scandinavian Journal of Economics*, 100, 2, 495-512.

Lohmann, S. (1992), «Optimal Commitment in Monetary Policy: Credibility versus flexibility», *American Economic Review*, 82, 273-286.

Muscattelli, A.; Trecroci, C. (2000), «Monetary policy rules, policy preferences, and uncertainty: recent empirical evidence», *Journal of Economic Surveys*, 14, 5.

O'flaherty, B. (1990), «The Care and Handling of Monetary authorities», *Economics and Politics*, March, 2, 1, 25-44.

Rogoff, K.; Sibert, A. (1988), «Elections and Macroeconomic Policy Cycles», *Review of Economic Studies*, January, 1-16.

Temple, J. (1998), «Central Bank Independence and Inflation: Good News and Bad News», *Economic Letters*, 61, 215-219.

Tietmeyer, H. (1991), «The Role of an Independent Central Bank in Europe», in P. Downes et R. Vaez-Zadeh eds., *The Evolving Role of Central Banks*, Washington: International Monetary Fund.

Waller, C. J. (1989), «Monetary policy Games and Central Bank politics», *Journal of money, credit and banking*, Nov., 21, 4, 422-432.

Waller, C. J.; Walsh, C. E. (1996), «Central Bank Independence, Economic Behaviour, and Optimal term lengths», *American Economic Review*, 86, 1, March, 150-167.





## Annex

Determination of the optimum level of conservatism of the team in charge of the forthcoming Monetary Policy Council

The loss function of Political Authority is given by (23)

$$E(L_t) = \frac{1}{2} \left[ \text{Var}(y_t) + \left( E(y_t) \right)^2 + \alpha' \beta' \text{Var}(\pi_t - \pi_{st}^*) \right] + \frac{1}{2} \left[ \alpha' \beta' \left( E(\pi_t - \pi_{st}^*) \right)^2 + \alpha' (1 - \beta') \text{Var}(\pi_t) + \alpha' (1 - \beta') \left( E(\pi_t) \right)^2 \right] \quad (23)$$

When shocks are not correlated, equation (23) reduces to:

$$E(L_t) = \frac{1}{2} \left[ \eta^2 \lambda^2 \text{Var}(\bar{\pi}) + \eta^2 \sigma_u^2 + \mu^2 \sigma_\varepsilon^2 + (\lambda - \mu)^2 \sigma_\delta^2 + \alpha' \beta' A^2 + \alpha' m^2 \lambda^2 \text{Var}(\bar{\pi}) + \alpha' m^2 \sigma_u^2 + \alpha' \sigma_\delta^2 \right]$$

In this expression, only  $m$  and  $\eta$  depend on  $\alpha$ .

$$\frac{\partial \left( E(L_t) \right)}{\partial \alpha} = \frac{\partial (\eta^2 + \alpha' m^2)}{\partial \alpha} \left( \lambda^2 \text{Var}(\bar{\pi}) + \sigma_u^2 \right) = 0 \text{ i.e.,}$$

$$\alpha = \lambda^2 \alpha'$$

The second order condition is:

$$\frac{\partial^2 \left( E(L_t) \right)}{\partial \alpha^2} (\alpha) > 0$$

One can easily verify that this condition is satisfied for  $\alpha = \lambda^2 \alpha'$

When shocks are correlated, equation (23) expands into a more complex expression. Despite an increased number of terms, this expression is still linearly related to  $m$  and  $\eta$ , both depending in turn on  $\alpha$ . The first order condition then results in a more complex homographic relationship between  $\alpha$  and  $\alpha'$ , defined as following:

$$\alpha = \frac{a\alpha' + b}{c\alpha' + d}$$

with

$$\begin{aligned} a &= \lambda(\lambda - \mu)^4 \text{Cov}(\bar{\pi}, \delta) + \lambda\mu(\lambda - \mu)^3 \text{Cov}(\bar{\pi}, \varepsilon) - (\lambda - \mu)^4 \text{Cov}(u, \delta) - \mu(\lambda - \mu)^3 \text{Cov}(u, \varepsilon) \\ b &= 2\lambda^3(1 - \lambda) \text{Cov}(\bar{\pi}, u) + \lambda^2(\lambda - \mu)^2 \text{Cov}(\bar{\pi}, \delta) - \lambda(\lambda - \mu)^2 \text{Cov}(u, \delta) + \lambda^5 \sigma_\pi^2 - \lambda^4 \sigma_\pi^2 + \lambda^3 \sigma_u^2 - \lambda^3 \mu \sigma_u^2 \\ c &= 2\lambda(\lambda - \mu) \text{Cov}(\bar{\pi}, u) - \lambda(\lambda - \mu)^2 \text{Cov}(\bar{\pi}, \delta) + \lambda\mu(\lambda - \mu) \text{Cov}(\bar{\pi}, \varepsilon) - (\lambda - \mu)^2 \text{Cov}(u, \delta) - \mu(\lambda - \mu) \\ &\quad \text{Cov}(u, \varepsilon) + \lambda^3 \sigma_\pi^2 - \lambda^2 \sigma_\pi^2 + \lambda \sigma_u^2 + \mu \sigma_u^2 \end{aligned}$$