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THE TEACHING MANUAL OF PEDRO NUNES

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While checking through sixty or so Latin miscellanies gathering dust in Evora's Public Library, I discovered among them a manuscript that contained the long-lost Portuguese original of Pedro Nunes' seminal and still relevant work, his *Libro de Algebra*, published in Spanish at Antwerp in 1567. This work was valued highly by most of the leading seventeenth century mathematicians, in England, France and Germany, as well as in Spain and Portugal, especially by John Wallis, Savilian professor of Geometry at Oxford University, and Jacques Peletier, Elias Vinet, Guillaume Gosselin and Simon Stevin in France. The Portuguese original has been searched for and theorized over for nearly four hundred years, as the basis of the most important of the many scientific publications by Nunes. Portugal's modern coinage portrays only one human, on its most attractive 100 escudos coin, not the usual royalty or politician, nor local fauna or flora, nor even the great discoverer, Vasco da Gama, but a bearded humanist, Pedro Nunes, who holds in his hands the globe that was first discovered and first mapped by Portuguese mariners, using nautical instruments and charts invented, or made more accurate, through his extraordinary mathematical genius.

The sixteenth century manuscript that contains the *álgebra* is an unpretentious, leather-bound miscellany of vellum, containing 214 folios in all. The first quire consists of thirty folios, or sixty pages, four of them blank. They have been trimmed to measure 15 cms wide by 21 cms high. Numbered 1-4 and 1-26, the first four contain a Church Calendar, followed by the *álgebra* (folios 1 recto to 12 recto) and a collection of Greek and Latin poems (folios 13 verso to 24 verso). These opening folios seem to have been written by Pedro Nunes himself; if not, in the handwriting of a sixteenth century humanist.

The next quire consists of ten folios of poorly written Portuguese prose,

covering various religious topics, but on smaller folios (14 cms by 19 cms), and written in six or more different hands. There follows a much later insert, a printed and illuminated copy of the *Epistola de Vasco Diaz de Frexenal*, on eight folios. The rest of the manuscript is filled with 170 folios of Latin prose, including many technical words and lengthy quotations in Greek, that cover the major topics of rhetoric. The page size of these final folios matches that of the first quire, and its small but neat lettering suggests a contemporary humanist's hand, but not Pedro's, it seems. However, the subject-matter may have been part of his lecture programme during his first years of teaching, as a lecturer in politics at the University of Lisbon. So far I have not had time to study these folios in detail.

On a Church Calendar that prefixes the *álgebra*, Easter is on April 13th, pointing to 1533, when Nunes was thirty one years old, and the mathematics tutor to the brothers of King John III and to some young noblemen, in the then royal city of Evora. What is surprising is the very close similarity in the titles, the contents and the formulae when one compares the 1533 teacher's manual with its 1567 Spanish version, greatly expanded for publication 34 years later. [This can be clearly seen on the Tables of Contents that have been circulated for this paper.] By 1533, Nunes had already translated most of the scientific works of Aristotle, Euclid and Ptolemy, and mathematical treatises by Arabic and Italian authors, and he had been appointed as the first Royal Cosmographer. It seems highly probable that his theories in geometry and algebra were by then well thought out, already sufficiently advanced for him to have a far better understanding of the workings of the globe, of cartography and of the differing shadows, as can be seen in his *Tratado da Sphera*, published in 1537, and his *De Crepusculis*, published in 1542. Only four years later he was also well able to refute solutions to apparently insoluble problems for the Ancient Greeks, as propounded by France's leading mathematician, Orontio Fineu (*De erratis Orontii Finaei*, 1546). The major addition from his thirty years of research and teaching Mathematics at the University of Coimbra was the collection of *problemas* (187 in all), in Part 3 of his Spanish *Algebra*, that could be of value to students of algebra today. I hope to have his original *algebra* published with an English translation, with the support of the Portuguese government, in 1992 or 1993.

When Nunes dedicated his *magnum opus* on algebra to Cardinal Henry, he dated it 1564, and located it not in Coimbra, but in Lisbon. It seems quite likely that at this stage the enlarged work was also written in Portuguese, like its introduction.

His original teaching materials had been expanded tenfold with the necessary elucidation of what he could explain by word of mouth, and with the 187 problems in the final chapter. These problems must have been developed while he was using his *álgebra* to teach his students at the University of Coimbra. Three years later, the book appeared in print, but in Spanish, and in Antwerp, by then a major commercial rival to Lisbon. The many Lusisms in the text clearly show that Spanish was not Pedro's native tongue, even though his wife came from Salamanca. Early in the dedication, Nunes refers to *nesta opulentissima cidade de Lixboa, onde tanto negotio ha desde extremo oriente e occidente, e ilhas do mar Oceano, e onde el Rey nosso senhor tem quarenta contadores de sua fazenda* ('this very rich city of Lisbon, to where so much commerce comes from the farthest Eastern and Western lands and islands of the Atlantic ocean, and where the King our Lord has 40 accountants for his treasury'). He then gives a strictly utilitarian justification for his *álgebra*, like those used by modern academics in applying for Government research and publication funds. *Por esta causa, he argues, vendo enquanto seia uil para ho uso dos homens esta arte que trata dos numeros e medidas* ('For this reason, I sell it as long as this art in numbers and measurements is useful for men to employ'). However, his choice of Spanish and of Antwerp, hopefully to give his *álgebra* greater publicity, proved to be disastrous. The busy port was overtly hostile to the Spanish, and no mecca for an abstract work on mathematics, and it seems that his text was only purchased by the professional mathematicians in Northern Europe.

Classical scholars may be interested in a collection of sixty Greek and Latin poems that follow the *álgebra*, in the same humanistic script, most of them eulogizing Pedro's Royal pupils and King. Two tease his fellow royal tutor, Antonio Pinheiro, for giving inflated marks to his pupils. After Philip II's take-over of Portugal, a conniving Pinheiro became its senior minister. Pedro's star pupil was Prince Henry, forced to give up his mathematical studies for theology, as he became Bishop, and then Archbishop and Cardinal – and an over-active Inquisitor General. Despite his Jewish blood, Nunes remained on very good terms with the Cardinal, dedicating his *Libro de Algebra* to him, perhaps as a substitute for all the mathematics classes he had missed. A most unusual feature of these epigrams is the fact that thirty six of them are in fluent Greek, a very rare and far from easy medium for poetry during the 16th century – except for someone who had tackled the very difficult Greek in Aristotle's scientific works. These skilfully composed poems will appear shortly in the local Classical journal, *Euphrosyne*.

There is a date on the first of the folios containing these poems that supports the 1533 deduced from the Calendar. On the top right-hand corner of the first page, one can see 1534, in the same ink and script as the main text. These dates fit in perfectly with those established in modern biographies of Pedro Nunes. From 1531 to 1534, he was tutoring the royal Princes and some noblemen at Évora, where it seems that he used the Calendar, and his Portuguese notes and equations for algebra, so as to plan the timetable for conducting his classes on mathematics. There he also began to compose his Elegiac poems in both Greek and Latin, while sharing accommodation with two inspiring Latin poets and brilliant Classical scholars, André de Resende and Nicolas Clenardo, both of them fellow tutors in Évora. By mid 1534, however, Nunes had left his friends in Évora, to become the first professor of an autonomous Mathematics department at the recently reformed University of Coimbra. While living there, after his retirement in 1552 from the Chair of Mathematics, Nunes returned once more to his poetry, and used it to bewail the death of his royal pupil, friend and generous benefactor, Prince Louis. In all, twenty two more poems in Latin and nine in Greek were added to his private *corpus* of twenty nine verse compositions. These dates can be backed up by the changing watermarks on the early and later folios. From folios 1 recto to 18 verso, a "right hand" can be seen, with its fingers only visible, beneath a five petal rosette, similar to early sixteenth century Genoese parchments. On folio 19 recto, however, a completely different watermark appears, the likely cut off point for his two groups of original poems.

It is also worth noting that Pedro's friend and one-time fellow royal tutor, André de Resende, had by then left Évora to join him in Coimbra, as Professor of Theology at the revitalized University, or perhaps at the richly endowed Royal College. There Resende delivered the prestigious *Oratio de Sapientia* on June 28th, 1551, to mark the anniversary of the foundation of the College by King John III. It is quite possible that he was still there in 1555, when Prince Louis died, on November 27th. Otherwise it is very likely that these two humanists kept in regular contact, both of them being closely linked to the Royal Court, including its final young King, Sebastian .

Theologians, finally, may be interested in the strong Catholic faith shown both by repeated religious symbols throughout the *álgebra* and the poems, and by five pages of commentary by Nunes on passages quoted from the New Testament. His favourite pupil and most generous benefactor was Prince Louis (next in line to

John III), who at one stage thought of becoming a Jesuit, and gave up his worldly goods and pleasures to live a truly pious life, but died prematurely in 1555. In the Évora manuscript, the date 1563 appears on two folios, 12 verso and 26 recto, and this seems the most likely time for his soul-searching notes on the Gospels, as he reached his sixties. I hope that his religious commentaries can also be published with the *álgebra*.

How much Sir Isaac Newton and later mathematicians owed to Pedro Nunes remains to be assessed. But there is no doubt as to the great importance of Nunes in the rapid development of algebra in Europe during the sixteenth and early seventeenth centuries, especially in Spain and Portugal. The importance of Mathematics in the modern world hardly needs to be stressed, and yet it seems that there is an increasing shortage of well-trained mathematics teachers, even in the more affluent countries. It is all the more creditable to the land of his birth that a sixteenth century mathematician, Pedro Nunes, holds pride of place on Portugal's most attractive and most valuable coin.

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