MPMM

Maintenance Performance
Measurement and Management

Proceedings of Maintenance Performance Measurement and Management (MPMM)

Conference 2014

4th & 5th SEPT 2014

COIMBRA

Department of Mechanical Engineering
Pólo II · FCTUC

A Control System Approach to Optimal Maintenance Planning for Building Retrofitting Project

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***Abstract*—Existing studies paid most attentions to the strategic planning of building retrofitting project. During the actual operation, the overall energy performance of the retrofitting project deteriorates over time owing to the usage and failures of the retrofitted facilities. For the sustainability of energy efficiency, the maintenance plan optimization for the retrofitted facilities must be considered, which suggests the possibility to sustain the building energy efficiency during operating. However, the overall energy performance reveals strong time dynamics subject to the failures and maintenance actions, which make the maintenance plan optimization a complicated problem. This paper characterizes the dynamics of the totality of the retrofitted facilities by the multi-state system models governed by a discrete-time Markov process. The impacts of both preventive and corrective maintenance actions on the over energy efficiency of the plant are then quantified by the present model. The optimization problem is thus casted into a stochastic optimal control problem which aims at maximizing the long-term energy efficiency and financial payback. An MPC based approach is employed to solve the present control problem. Finally, a case study is conducted to demonstrate the effectiveness of the present approach.**

***Keywords*—Building retrofitting, Facilities maintenance policy, Energy efficiency, Multi-state system, Control system**

I. Introduction

Existing studies on improving building energy efficiency mainly focus on two aspects: the strategic planning of the building retrofitting [1], [2], [3], [4], and the optimal operating of a certain kind of facilities in the building [5], [6], [7]. However, after the implementation of an energy efficiency option or a retrofitting plan, the energy efficiency of the retrofitted facilities can not escape from deterioration due to the usage and the failures. From the management perspective, the overall energy efficiency of the plant degrades over time, as facilities can contribute less energy savings than people expected due to the age or usage, and breakdowns can take place which completely stop items from contributing energy saving. For the sustainability of building energy efficiency, maintenance as an important aspect of the facilities management, deserves more attention from the energy efficiency community.

However, the optimal maintenance plan for a building retrofitting project is not easy to achieve. The totality of the retrofitted facilities can demonstrate significant time dynamics under the impacts of both failures and maintenance actions on the management level. The energy efficiencies of the retrofitted facilities can either degrade due to the usage and failures or be restored by the maintenance actions, and the overall energy performance of the retrofitted plant over a period of time can be influenced by various maintenance actions on different facilities at different time instants. The maintenance decision can be difficult due to this complexity. Furthermore, economy must be taken into account by the maintenance planner as well. The maintenance budgets are often limited in practice, and the cost-effectiveness of the maintenance actions, as a part of the economy of retrofitting project, should not be ignored.

The control system approach can be a good perspective to address the complexity of the maintenance planning optimization. Control system approaches have been used to obtain the optimal maintenance strategies of single-unit or multi-components machines from the manufacturing industry [8], [9]. In these literatures, the studied machines are assumed to have multiple working states and a failure state. The productivity of the machine differs under different working modes and the failure mode, i.e., the different system states. The state transition of the machine is governed by a continuous-time Markov process, and the influence of maintenance actions, including Preventive Maintenance (PM) and Corrective Maintenance (CM) are represented by the jump rates from deteriorated of failure states to better working states. The multi-state systems are also popular among the reliability community. Literatures [10], [11] focused on the modeling of deteriorating systems using the multi-state system approach. The corresponding reliability optimization under realistic conditions are also investigated in these literatures. However, none of these studies, either from the control science community or from the reliability engineering community, has ever considered a plant that can not be modeled by one single multi-state system.

In this paper, the totality of retrofitted facilities in the building energy efficiency context, instead of individual items, is considered as our plant. Multiple categories of heterogeneous facilities are involved in the retrofitting project, each corresponds to a multi-state system model. The possible working modes and jump rates can be different between the categories. The maintenance actions are planned on the management level, i.e., for a group of homogenous facilities rather than for a single item. As a consequence, the numbers of items under different working modes, rather than the probabilities of one single item being under different modes, are the main concerns of the present approach. The dynamics of number of items under different states is governed by a discrete-time Markov process, due to the limitation of inspection capability in practice. The maintenance actions over each sampling period are also represented by the jump rates of the Markov process, which allow the growth of the item population under better working mode and the decrease of the item population under worse working mode or failure mode. By the employment of multi-state system models in the present model, the impact of the PM and CM actions on the overall energy efficiency of the plant can be quantified. The maintenance plan optimization is thus transformed into a stochastic optimal control problem taking into account several different multi-state system models and the dynamics of item populations under respective system states. Two objectives are introduced for the optimal control problem: the maximum of overall energy saving and the maximum of payback ratio of the retrofitting project over a pre-decided time period. A Model Predictive Control (MPC) based approach is employed to solve the present optimal control problem. A simple practical retrofitting project is adopted as the case study to test and verify the feasibility of the present approach.

The remainder of the paper consists of four sections. Section 2 gives the modelling of the multi-state systems and the stochastic optimal control problem formulation. Section 3 introduces the MPC based approach. Section 4 provides the simulation results and analysis of the case study. Section 5 draws conclusion and discusses future research.

II. Control Problem Modeling

A. Problem statement

There are generally two types of facilities involved in the context of the building energy retrofitting project. Type-I facilities are generally considered as single-unit systems and do not undertake any preventive maintenance over the life-cycle. After the breakdown of an item from type-I facilities, corrective replacement take place and the failed item is scrapped. Globes, motion sensors, desktop PC monitors are typical type-I facilities. Type-II facilities are more complicated than type-I facilities, which are often considered as multi-component systems. Preventive maintenance can sustain the energy performance of type-II facilities. Failures that are repairable by corrective maintenance can take place during the operation of the item from type-II facilities before the scrap of the item. Air-conditioner and heat pumps are typical type-II facilities. As the lifespan of type-II facilities are usually as long as over ten years, the preventive/corrective replacement for type-II facilities is not taken into account in the present study.

The energy performances of type-I facilities are considered constant over the life-cycle in the present study. For type-II facilities, the energy performance will degrade as usage over life-cycle. By the implementation of preventive and corrective maintenance actions, the breakdowns and the performance degradation of the items are removed, i.e., the deterioration of the facilities has been controlled. The models which characterize the controlled deterioration of the two types of facilities are given in the following section:

B. Controlled process of type-I and type-II retrofitted facilities deterioration

Let N_I denote the total number of a group of type-I facilities. Type-I facilities have two modes denoted by $S_I = \{1, 2\}$. Mode 2 means the item is not available. Items under mode 2 do not contribute to the energy saving of the retrofitting project. Mode 1 denotes the normal working status of the item. The energy saving of the item is constant value under mode 1. During practical operation, an item has a possibility to change from one mode to another. According to [8], the transition between the modes of the item can be governed by a discrete-time Markov process $\{r(t_k), k \geq 0\}$, where t_k denotes the sampling instants. The transition matrix taking value in S_I can be obtained:

$$Q_I(t_k) = \begin{pmatrix} q_{11}^I(t_k) & q_{21}^I(t_k) \\ q_{12}^I(t_k) & q_{22}^I(t_k) \end{pmatrix}$$

where $q_{ij}^I(t_k) \geq 0, j \neq i, q_{ii}^I(t_k) = 1 - \sum_{j \neq i} q_{ij}^I(t_k)$. Jump rate $q_{21}^I(t_k)$ represents the influence of corrective replacement denoted by $\omega_r^I(t_k)$, i.e., the control rate. $q_{12}^I(t_k)$ represents the deterioration rate of the population of items under mode 1. The transition matrix of the controlled Markov process can be rewritten as:

$$Q_I(t_k, \omega_r^I(t_k)) = \begin{pmatrix} 1 - q_{12}^I(t_k) & \omega_r^I(t_k) \\ q_{12}^I(t_k) & 1 - \omega_r^I(t_k) \end{pmatrix}$$

Let $x^I(t_k) = [x_1^I(t_k) \ x_2^I(t_k)]^T$ denote the number of items under mode 1 and 2 at instant t_k, where $x_1^I(t_k) + x_2^I(t_k) = N_I$, the states of the group of type-I facilities at next sampling instant can be obtained:

$$x^I(t_{k+1}) = Q_I(t_k, \omega_r^I(t_k)) x^I(t_k). \quad (1)$$

Type-II facilities have four modes denoted by $S_{II} = \{1, 2, 3, 4\}$, where mode 4 means the item is under repair and not available, mode 1,2,3 denote there working modes of the item: good, average and bad. Item under mode 1 has the best energy performance, i.e., provides largest energy saving. Item under mode 2 and 3 only provides discounted energy savings compared with Item under mode 1. For the simplicity, the energy performances from mode 1,2 and 3 are considered constant. The transition matrix taking value in S_{II} can be obtained:

$$Q_{II}(t_k) = \begin{pmatrix} q_{11}^{II}(t_k) & q_{21}^{II}(t_k) & q_{31}^{II}(t_k) & q_{41}^{II}(t_k) \\ q_{12}^{II}(t_k) & q_{22}^{II}(t_k) & 0 & 0 \\ 0 & q_{23}^{II}(t_k) & q_{33}^{II}(t_k) & 0 \\ q_{14}^{II}(t_k) & q_{24}^{II}(t_k) & q_{34}^{II}(t_k) & q_{44}^{II}(t_k) \end{pmatrix}$$

where $q_{ij}^{II}(t_k) \geq 0, j \neq i, q_{ii}^{II}(t_k) = 1 - \sum_{j \neq i} q_{ij}^{II}(t_k)$. Jump rates $q_{21}^{II}(t_k)$ and $q_{31}^{II}(t_k)$ are the preventive maintenance rates denoted by $\omega_{pa}^{II}(t_k)$ and $\omega_{pb}^{II}(t_k)$. Jump rate $q_{41}^{II}(t_k)$ is the corrective maintenance rate denoted by $\omega_r^{II}(t_k)$. $\omega_{pa}^{II}(t_k)$, $\omega_{pb}^{II}(t_k)$ and $\omega_r^{II}(t_k)$ are the control rates. Let $\omega_{II}(t_k) =$

$[\omega_{pa}^{II}(t_k)\ \omega_{pb}^{II}(t_k)\ \omega_r^{II}(t_k)]$. Similarly, the transition matrix of the controlled Markov process can be rewritten as:

$$Q_{II}(t_k, u_{II}(t_k)) = \begin{pmatrix} q_{11}^{II}(t_k) & \omega_{pa}^{II}(t_k) & \omega_{pb}^{II}(t_k) & \omega_r^{II}(t_k) \\ q_{12}^{II}(t_k) & q_{22}^{II}(t_k) & 0 & 0 \\ 0 & q_{23}^{II}(t_k) & q_{33}^{II}(t_k) & 0 \\ q_{14}^{II}(t_k) & q_{24}^{II}(t_k) & q_{34}^{II}(t_k) & 1-\omega_r^{II}(t_k) \end{pmatrix}$$

where $q_{12}^{II}(t_k)$ and $q_{23}^{II}(t_k)$ represent the degradation of the energy performance, $q_{14}^{II}(t_k)$, $q_{24}^{II}(t_k)$ and $q_{34}^{II}(t_k)$ represent the deterioration of the population of working items. Let N_{II} denote the total number of the type-II items, $x^{II}(t_k) = [x_1^{II}(t_k)\ x_2^{II}(t_k)\ x_3^{II}(t_k)\ x_4^{II}(t_k)]^T$ denote the number of items working under mode 1,2,3 and 4 respectively at instant t_k, $\sum_{i=1}^4 x_i^{II}(t_k) = N_{II}$. The states of the group of type-II items at next sampling instant can be obtained:

$$x^{II}(t_{k+1}) = Q_{II}(t_k, \omega_{II}(t_k))x^{II}(t_k). \quad (2)$$

The jump rates in $Q_I(t_k)$ and $Q_{II}(t_k)$ can be characterized by different deterioration models from reliability engineering [12]. In the present models, due to the lack of extensive studies on the retrofitted facilities, several assumptions are made to figure out the jump rates:

1) The jump rates are considered constant over time;
2) The failure rate of the items is independent of the working modes, i.e., $q_{i4}^{II}(t_k)$ with $i = 1,2,3$ are the same values decided only by the Mean Time Between Failures (MTBF);
3) For type-II facilities, the average running times from mode good to average and from average to bad are prior known, denoted by $t1$ and $t2$. Deterioration rates $q_{12}^{II}(t_k)$ and $q_{23}^{II}(t_k)$ are thus characterized by $t1$ and $t2$;
4) The maintenance actions take place prior to the deterioration of item populations. Over the sampling period when maintenance actions are applied, only the items that are not maintained can deteriorate.

An exponential decay model from [12] is thus adopted to decide the jump rates. Let θ denote the MTBF of a facility and $k = \theta^{-1}$, the failure rate is $1-e^{-k}$, i.e., the value of jump rate $q_{12}^{I}(t_k)$ for type-I facilities and jump rates $q_{i4}^{II}(t_k)$ with $i = 1,2,3$ for type-II facilities. Similarly, $q_{12}^{II}(t_k) = 1-e^{-t1^{-1}}$ and $q_{23}^{II}(t_k) = 1-e^{-t2^{-1}}$.

C. *Characterization of energy and financial performances*

In the present model, the energy and financial performance characteristics are required to evaluate the energy efficiency and cost effectiveness of a maintenance plan. Let T denote the length of time period that the planning covers, namely evaluation period. The total energy saving and the payback ratio over the evaluation period $[0,T]$ are the main required characteristics, subject to a series of constraints, including the targeted energy saving limits, the budget limits and non-negative NPV limits.

Given n_I groups of type-I facilities and n_{II} groups of type-II facilities with respective rated lifespan and deterioration characteristics. Let $x(t_k) = [x_I(t_k)\ x_{II}(t_k)]^T$ denote the system states inspected at instant t_k, where $x_I(t_k) = [x_i|_n^I]^T$, $n \in \{1,2\}, i \in [1,n_I]$; $x_{II}(t_k) = [x_i|_n^{II}]^T$, $n \in \{1,2,3,4\}, i \in [1,n_{II}]$. For the energy conservatism, $x(t_k)$ is considered to be the system states over sampling period $[t_{k-1},t_k]$. The control variables, i.e., the maintenance actions, are thus represented by $u(t_k) = [u_I(t_k)\ u_{II}(t_k)]^T$, where $u_I(t_k) = [\omega_i|_r^I(t_k)x_i|_2^I(t_k)]^T$, $i \in [1,n_I]$; $u_{II}(t_k) = [\omega_i|_{pa}^{II}(t_k)x_i|_2^{II}(t_k)\ \ \omega_i|_{pb}^{II}(t_k)x_i|_3^{II}(t_k)\ \ \omega_i|_r^{II}(t_k)x_i|_4^{II}(t_k)]^T$, $i \in [1,n_{II}]$. The control actions are considered to be implemented over the next sampling period $[t_k,t_{k+1}]$.

The performance characteristics are formulated as following: let $a_i^I(t_k) = [a_i^I|_n(t_k)]$, $n \in \{1,2\}, i \in [1,n_I]$; $a_i^{II}(t_k) = [a_i^{II}|_n(t_k)]$, $n \in \{1,2,3,4\}, i \in [1,n_{II}]$. $a_i^I(t_k)$ and $a_i^{II}(t_k)$ denote the average energy savings that an item contributed under different working modes over the sampling period $[t_k,t_{k+1}]$. Similarly, $b_i^I(t_k)$ and $b_i^{II}(t_k)$ denote the average cost savings. Apparently, $a_i^I|_2(t_k)$, $b_i^I|_2(t_k)$, $a_i^{II}|_4(t_k)$ and $b_i^{II}|_4(t_k)$ are constantly 0. Let $C_i^I = [C_i^I|_r], i \in [1,n_I]$, $C_i^{II} = [C_i^{II}|_{pa}, C_i^{II}|_{pb}, C_i^{II}|_r], i \in [1,n_{II}]$, where $C_i^I|_r, C_i^{II}|_{pa}, C_i^{II}|_{pb}, C_i^{II}|_r$ denote the respective maintenance costs. The energy saving at each instant t_k and the overall energy saving over $[0,T)$ are thus obtained:

$$\begin{cases} ES(t_k) = \sum_{i=1}^{n_I} a_i^I(t_k)x_i^I(t_k) + \sum_{i=1}^{n_{II}} a_i^{II}(t_k)x_i^{II}(t_k), \\ ES|_{all} = \sum_{k=0}^{K_T} ES(t_k), \end{cases} \quad (3)$$

and the accordingly cost savings are obtained:

$$\begin{cases} B(t_k) = \sum_{i=1}^{n_I} b_i^I(t_k)x_i^I(t_k) + \sum_{i=1}^{n_{II}} b_i^{II}(t_k)x_i^{II}(t_k), \\ B|_{all} = \sum_{k=0}^{K_T} B(t_k), \end{cases} \quad (4)$$

the maintenance cost at each time instant is obtained:

$$\begin{aligned} h(t_k) = &\sum_{i=1}^{n_I} C_i^I|_r u_i|_r^I(t_k) + \sum_{i=1}^{n_{II}} C_i^{II}|_{pa} u_i|_{pa}^{II}(t_k) + \\ &\sum_{i=1}^{n_{II}} C_i^{II}|_{pb} u_i|_{pb}^{II}(t_k) + \sum_{i=1}^{n_{II}} C_i^{II}|_r u_i|_r^{II}(t_k), \end{aligned} \quad (5)$$

and the overall investment of the retrofitting project:

$$h|_{all} = h_0 + \sum_{k=1}^{K_T} h(t_k), \quad (6)$$

where h_0 denotes the initial investment of the retrofitting project, K_T denote the number of sampling instants over $[0,T]$.

The profit of the project is then obtained by $P = B|_{all} - h|_{all}$. However, the more usual method to evaluate the economy of a project is NPV. The NPV over $[0,T]$ is:

$$NPV = \sum_{k=1}^{K_T} \frac{B(t_k) - h(t_k)}{(1+d)^k} - h_0, \quad (7)$$

where d denotes the discount rate for NPV calculation.

D. *Control Problem Formulation*

Based on the equations obtained in the previous sections, the population dynamics of a retrofitting project is characterized

ISBN 978-972-8954-42-0 | http://dx.doi.org/10.14195/978-972-8954-42-0_10

by the following equation:

$$x(t_{k+1}) = f(x(t_k), u(t_k)) \\ = [Q_I(t_k, \omega_r^I(t_k))x^I(t_k), Q_{II}(t_k, \omega_{II}(t_k))x^{II}(t_k)]^T, \quad (8)$$

where $x(t_0) = x_0$.

The control objective is to find a control law $\mathbf{u}(\cdot) = \{(u(t_k), \omega(t_k), k \in [0, K_T]\}$ with $\omega(t_k) = \{\omega_r^I(t_k), \omega_{pa}^{II}(t_k), \omega_{pb}^{II}(t_k), \omega_r^{II}(t_k)\}$, which minimize the following performance index:

$$J(x_0, \mathbf{u}(\cdot)) = E[-\lambda_1 ES|_{all} + \lambda_2 \frac{h|_{all}}{B|_{all}}], \quad (9)$$

subject to

$$\begin{cases} ES|_{all} \geq \alpha, \\ \sum_{k=i*M+1}^{(i+1)*M} h(t_k) \leq \beta, \ i = 0, 1, 2, 3, ... \\ NPV \geq 0, \\ \omega_{pa}^{II}(t_k), \omega_{pb}^{II}(t_k) = 0, \ k \notin P, \\ \omega_r^I(t_k), \omega_r^{II}(t_k) = 0, \ k \notin Q, \end{cases} \quad (10)$$

where λ_1 and λ_1 denote the weighting factors. α denotes the target energy saving amount, β denotes the maintenance budget limit over a series of fixed time periods $[t_{i*M+1}, t_{(i+1)*M}]$, $i = 0, 1, 2, 3, ...$, with constant number of sample instants M. P denotes a set of time instants, indicating when the preventive maintenance actions are scheduled to take place. Similarly, Q includes the time when the corrective maintenance actions are scheduled to take place.

III. MPC APPROACH

To solve the optimal control problem represented by equations (9) and (10), an MPC based approach is adopted. In MPC approaches, an open loop optimal control problem is repeatedly solved over a finite horizon according to the plant model prediction. The obtained optimal open loop control is then used to generate the optimal control input for the problem to be solved, with which the state variables executed over the next finite horizon are obtained. As the optimal controller over the next finite horizon is actually a function of the system state from the previous control step, a closed-loop feedback is thus obtained. Consider a horizon with length N, a mathematical transformation of the optimal control problem is applied, and the open loop optimal control problem over $[t_m, t_{m+N}]$ is accordingly defined as the following minimization problem:

$$min \ J'(x(t_m), \mathbf{u}'|_m(\cdot)) = E[-\lambda_1 ES'|_m + \lambda_2 \frac{h'|_m}{B'|_m}], \quad (11)$$

subject to

$$\begin{cases} ES'|_m \geq \alpha', \\ h'|_m \leq \beta', \ m \in R \\ NPV'|_m \geq h'_0, \\ \omega_{pa}^{II}(t_k), \omega_{pb}^{II}(t_k) = 0, \ k \notin P, \\ \omega_r^I(t_k), \omega_r^{II}(t_k) = 0, \ k \notin Q, \end{cases} \quad (12)$$

where

$$ES'|_m = \sum_{k=m+1}^{m+N} ES(t_k), \quad (13)$$

$$h'|_m = \sum_{k=m+1}^{m+N} h(t_k), \quad (14)$$

$$B'|_m = \sum_{k=m+1}^{m+N} B(t_k), \quad (15)$$

$$NPV'|_m = \sum_{k=m+1}^{m+N} \frac{B(t_k) - h(t_k)}{(1+d)^k}, \quad (16)$$

α', β' denote the proportional targeted energy saving amount and maintenance budget over $[t_m, t_{m+N}]$, h'_0 denotes a proportion of initial investment h_0 that is expected to be covered by the cash flow over $[t_m, t_{m+N}]$.

This problem is solved over the interval $[t_m, t_{m+N}]$ when $m \in P$ or $m \in Q$, and a series of optimal control rates are obtained, represented by $\boldsymbol{\omega}'|_m = \{\omega'|_m(t_k) : k = m, m+1, ..., m+N-1\}$. For the sake of easy implementation, a DE based approach is thus applied to solve problem (11) [13]. Only the optimal solution in the first sampling period $[t_m, t_{m+1}]$ is applied, represented by $\bar{\boldsymbol{\omega}}|_m = \{\omega'|_m(t_m)\} = \{\bar{\omega}|_m(x(t_m))\}$, where the last equation is to emphasize the functional dependence of the optimal control on the initial state $x(t_m)$ of the MPC formulation in equations (11)-(16). According to equation (8), $\bar{\boldsymbol{\omega}}|_m$ is applied, $u(t_m)$ and $x(t_{m+1})$ are thus obtained. $x(t_{m+1})$ then becomes the initial condition of the MPC formulation over the time horizon $[t_{m+1}, t_{m+N+1}]$. When $m \notin P$ and $m \notin Q$, the control rates $\boldsymbol{\omega}(t_m) = 0$ is implemented as a solution. These are taking place consecutively over the control period to obtain the optimal control rates $\bar{\boldsymbol{\omega}}$. $x(t_k)$ is then applied as the initial state for the open loop optimal control problem over the next finite horizon. In summary, the following MPC algorithm can thus be formulated [14]:

A. *MPC Algorithm*

Initialization: Let initial state $x(t_0) = x_0$ and $m = 0$.
(i) Compute the open loop optimal solution $\{\omega'|_m(t_k)\}$ of the problem formulation (11)-(16), where $k = m, m+1, ..., m+N-1$.
(ii) The MPC controller $\bar{\boldsymbol{\omega}}|_m = \{\omega'|_m(t_m)\}$ is applied to the plant in the sampling interval $[m, m+1]$. The remains of the open loop optimal solution $\{\omega'|_m(t_k) : k = m+1, ..., m+N-1\}$ are discarded. $x(t_{m+1})$ are then obtained according to:

$$x(t_{m+1}) = f(x(t_m), u(t_m)) \\ = [Q_I(t_m, \omega_r^I(t_m))x^I(t_m), Q_{II}(t_m, \omega_{II}(t_m))x^{II}(t_m)]^T$$

and executed over the period $[t_m, t_{m+1}]$.
(iii) Let $m := m+1$ and go back to step (i).

Due to the constraint $\omega(t_m) = 0$, $m \notin P$ and $m \notin Q$, it is not necessary to solve the open loop optimal control problem over $[t_m, t_{m+N}]$, and $x(t_{m+1})$ is obtained by $x(t_{m+1}) = f(x(t_m), 0)$. The above MPC algorithm will go over the control period to solve out the optimal control strategy.

IV. Simulation And Verification

A. Case study

A small retrofitting project for a government office building is presented as our case study to verify the effectiveness of the present model. The retrofitting plan is decided prior to the maintenance planning, thus the objective of our case study is to obtain the optimal maintenance plan over a pre-decided implementation period. There are 5 types of facilities involved in the retrofitting, listed in Table I. In this table, the quantities represent the number of retrofitted facilities from each category. At the initial stage, all facilities are in good condition. The unit prices represent the initial investment taking into account all the purchase and installation. The unit energy savings and cost savings are the average measure over one year. During the implementation period, the unit savings are considered constant. The corrective cost, preventiveA cost and preventiveB cost represent the average costs of implementing the respective maintenance actions, where preventiveA refers to the preventive maintenance action that restores the system state from average to good, and preventiveA refers to the one that restores the system from bad to good. Preventive maintenance does not work on the type-I facilities. The MTBF, the mean time from good to average and the mean time from average to bad are given in Table II.

The maintenance plan is scheduled to be implemented over 120 months. From the auditing, the energy baseline of this retrofitting project is known as 2,249,500 kWh per year. The targeted energy saving amount is 10% of the energy baseline over the maintenance plan implementation period, which is 2,249,500 kWh. The yearly maintenance budget is $15,000. The discount rate for NPV calculation taking into account the interests and inflation is 1.77% per year. An inspection will be applied at the end of each month to monitor the status of the retrofitted facilities, i.e., the sampling instants and sampling periods. The maintenance schedule is thus pre-decided as the following: the preventive maintenance actions take place at the end of every year, i.e., the end of month 12, 24, 36,..., the corrective maintenance take place at the end of every seasons, i.e., the end of month 3, 6, 9,...Therefore, in our case study, $P = \{12, 24, 36, 48, ..., 120\}$, $Q = \{3, 6, 9, 12, ..., 120\}$ and $M = 12$.

B. illustrative results and analysis

Table III illustrates the performance characteristics in three different maintenance contexts: without maintenance, full

TABLE II
Mean times of involved retrofitted facilities

Facilities	Type	MTBF	t1 (good to average)	t2 (average to bad)
Motion sensor	I	33.5	N/A	N/A
35W retrofit ECG	I	27.2	N/A	N/A
180W new projector	I	32.8	N/A	N/A
3kW heat-pumps	II	52	25.2	100
Latest airconditioner	II	43.8	21.6	86.4

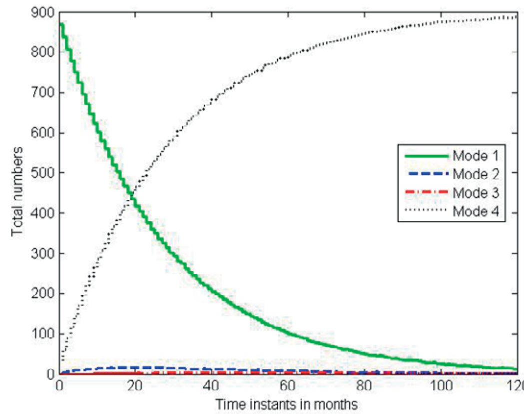

Fig. 1. Time dynamics of the totality of retrofitted facilities in the No Maintenance context

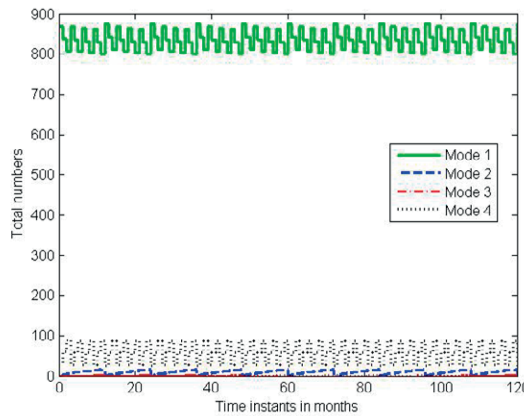

Fig. 2. Time dynamics of the totality of retrofitted facilities in the Full Maintenance context

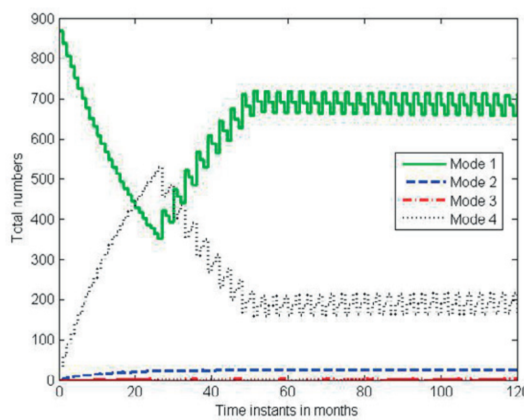

Fig. 3. Time dynamics of the totality of retrofitted facilities in the Optimal Maintenance 1 context

TABLE I
Characteristics of involved retrofitted facilities

Facilities	Type	Quantities	Unit Price ($)	Unit Energy Saving (kWh)	Unit Cost Saving ($)	Corrective Cost ($)	PreventiveA Cost ($)	PreventiveB Cost ($)
Motion sensor	I	125	196	1141	135.1	196	N/A	N/A
35W retrofit ECG	I	682	14.19	102	10.91	14.19	N/A	N/A
180W new projector	I	48	490.8	230.4	25.95	263.28	N/A	N/A
3kW heat-pumps	II	11	1250	8640	973.3	201	47	65
Latest airconditioner	II	36	989	1782	195.65	175	26	35

TABLE III
Performance characteristics of obtained maintenance plan in different contexts

Contexts	Energy Saving (kWh)	Ratio	NPV ($)	PaybackPeriod (months)	Investment ($)	Maintenance Cost ($)	Profit ($)
No Maintenance	1093608	4.78%	29346.88	48.69	107090	0	34395.83
Full Maintenance	3653566	15.98%	125981.1	52.81	293923.6	186833.6	144914.3
Optimal Maintenance 1	2580266	11.28%	153002.4	40.86	178613.4	71523.37	172466.3
Optimal Maintenance 2	3563632	15.58%	188545.3	41.35	243243.1	136153.2	212668.4
Optimal Maintenance 3	2559074	11.19%	161228.9	38.89	164281.5	57191.47	181392.5

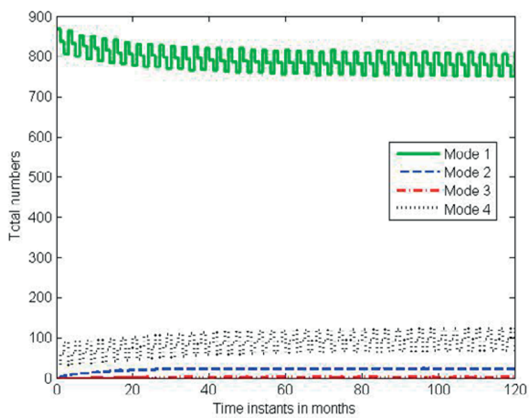

Fig. 4. Time dynamics of the totality of retrofitted facilities in the Optimal Maintenance 2 context

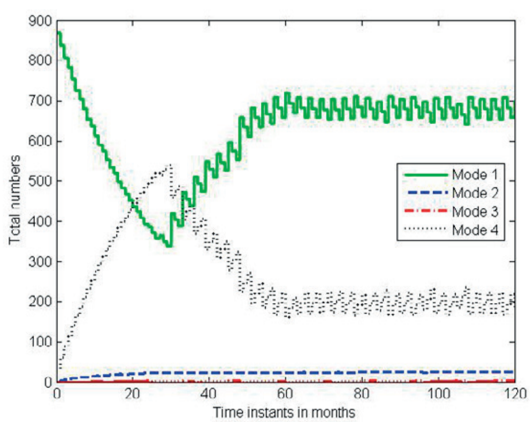

Fig. 5. Time dynamics of the totality of retrofitted facilities in the Optimal Maintenance 3 context

maintenance and optimal maintenance. The performance characteristics without maintenance illustrates the impact of deterioration to the plant. The full maintenance policy allows all the degraded or failed items to be restored to good states without taking budget into account. The optimal maintenance context applies optimal maintenance plan obtained by the present approach. In Table III, it can be observed that without maintenance, the energy efficiency of the plant can not sustain against the deterioration. The economy of the retrofitting project is seriously damaged as well. This reveals the importance of maintenance for a retrofitting project from both energy efficiency and cost-effectiveness perspectives. The full maintenance plan obtains the maximum energy saving amount among all the contexts, but the corresponding financial performance appears unacceptable, due to the longest payback period and the lowest profit and NPV. Furthermore, the maintenance expenditure of the full maintenance plan considerably exceeds the maintenance budget, which is $150,000 in ten years. The performance of full maintenance plan reveals the necessity of maintenance plan optimization.

The following three rows illustrate the performance of the optimal maintenance plan obtained by the present approach. Different combination of weighting factors are employed in these contexts: for optimal maintenance 1, $\lambda_1 = 0.5$ and $\lambda_2 = 0.5$, for optimal maintenance 2, $\lambda_1 = 0.75$ and $\lambda_2 = 0.25$, for optimal maintenance 3, $\lambda_1 = 0.25$ and $\lambda_2 = 0.75$. These different combinations show different emphasis on the multiple objectives: optimal maintenance 1 considers the balance between two objectives, optimal maintenance 2 reveals stronger need for the energy efficiency and optimal maintenance 3 emphasizes the financial payback of the retrofitting project. This allow the decision makers to use the present approach to obtain maintenance plan subject to their specific requirements.

Figs. 1 - 5 demonstrate the time dynamics of the totality of retrofitted facilities in respective contexts. In all these figures, the solid curves illustrates the dynamics of the totality of items under working mode 1, i.e., the good state. The dashed curve illustrates the dynamics of the total number of the items that are not available, comprising the items under mode 2 from type-I facilities and mode 4 from type-II facilities. The dashdoted curve and dotted curves show the dynamics of the totality of items from type-II facilities that are under working mode 2 and 3, respectively.

V. Conclusion

This paper employs a multi-state system model that is governed by a discrete-time Markov process to characterize the deterioration of the totality of retrofitted facilities from a building retrofitting project in the energy efficiency context. Different from the existing multi-state system approaches in literatures, the present model takes into account heterogeneous categories of retrofitted facilities. Each category is modeled by a multi-state system, and the involved multi-state systems have different states and jump rates. The totality of the groups of heterogeneous facilities instead of individual items is adopted as our plant and the main concern of maintenance plan optimization. The employment of multi-state system in the present model allows the quantified evaluation of the impact of preventive and corrective maintenance actions over a long period of time on the overall energy efficiency of the plant. The optimization problem is accordingly casted into a stochastic optimal control problem, by which the significant time dynamics of the totality of the retrofitted facilities can be address by a control system approach. An MPC based approach is thus presented to solve the stochastic control problem. A simple practical retrofitting project adopted as the case study is investigated to verify the effectiveness of the present approach. From simulation results, the effectiveness of the present approach can be observed. Furthermore, the present approach allows the decision makers to obtain optimal maintenance plan taking into account their specific requirements.

The present work calls for further studies on the following topics: the introduction of maintenance polices subject to more realistic conditions; the employment of more practical multi-state system models and deterioration models; the further studies on control system approaches to solve the proposed stochastic optimal control problem.

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