

Societe Platonicienne Internationale Associazione Internazior dei Platonisti

Sociedad Internacional de Platonistas

Internationale Platon-Gesellscha

Imprensa da Universidade de Coimbra

> Coimbra Jniversiy Press

# Movάς and ψυχή in the *Phaedo*

shows that the argument for the final proof is better than previously thought. Such an interpretation of the final proof lends credence to Platonic intermediates.

Keywords: soul, number, the Odd, the Even, unit, immortality, *Phaedo*, intermediates

## Sophia Stone

Lynn University SStone@lynn.edu

# ABSTRACT

The paper analyzes the final proof with Greek mathematics and the possibility of intermediates in the Phaedo. The final proof in Plato's Phaedo depends on a claim at 105c6, that  $\mu o \nu \alpha \varsigma$ , 'unit', generates περιττός 'odd' in number. So, ψυχή 'soul' generates  $\zeta \omega \eta$  'life' in a body, at 105c10-11. Yet commentators disagree how to understand these mathematical terms and their relation to the soul in Plato's arguments. The Greek mathematicians understood odd numbers in one of two ways: either that which is not divisible into two equal parts, or that which differs from an even number by a unit. (Euclid VII.7) Plato uses the second way in the final proof. This paper argues that a proper understanding of these mathematical terms within Greek mathematics

At the end of 'Equals and Intermediates in Plato', John Rist concludes that the attempts to show Plato held a doctrine of intermediates in the dialogues should be suspect, yet admits that in some passages, 'Plato appears to discuss a plurality of non-sensible  $\mu ov \dot{\alpha} \delta \varepsilon c$ .'' While it may prove problematic to uncover a 'doctrine' of intermediates in the dialogues, it is equally problematic to understand his arguments without attention to Plato's use of mathematical concepts. I explore Rist's suggestion by analyzing three passages in the *Phaedo*, that non-sensible units may indicate intermediates, mathematical objects between Forms and sensible objects<sup>2</sup>.

The Greek conception of ἀριθμός, 'number' as a limited multitude (Euclid VII.2) is crucial for understanding Plato's use of sensible and non-sensible multitudes in the argument for the final proof of the immortality of the soul.<sup>3</sup> The final proof relies on an analogy between the presence of life in the soul determining the soul's immortality and the presence of μονάς in an ἀριθμός determining three's being odd. The analogy, as I understand it, is this: just as  $\mu ov \alpha \varsigma$  is a sign that a number is odd,  $\psi v \chi \dot{\eta}$  is a sign that a body is alive. The Greek mathematicians understood odd numbers in one of two ways: either that which is not divisible into two equal parts, or that which differs from an even number by a unit. (Euclid VII.7) Plato in the Phaedo at 105c6 is clearly using the latter definition. We should apply this second definition to the analogy in the final proof: the left over monad in odd ἀριθμοί is analogous to the life bearing soul that makes a composite body and soul alive.

In Greek mathematics, one of the limits in an  $\dot{\alpha}\rho\iota\theta\mu\dot{\alpha}\varsigma$  is how it can be divided, whether that division is equal or unequal, these divisions were called  $\dot{\alpha}\rho\tau\iota\alpha\varsigma$  'even' and  $\pi\epsilon\rho\iota\tau\tau\dot{\alpha}\varsigma$  'odd,' respectively. This is why Plato understood the unit 'left over' from an equal division of multitudes to entail necessarily that the multitude is "odd." (Euclid VII.7) We can see why Plato would use Greek mathematical notions to argue for the immortality of the soul. Just as there is nothing intrinsic that makes a body alive, there is nothing intrinsic to a number — only to an even one — that makes it have an equal collection of units. Yet there is a necessary connection with a soul's participation in the Form Life, which makes the soul 'alive', just as there is a necessary connection with a unit's participation in the Form Odd, which makes the unit 'odd'. This is how the 'safe' hypothesis at 100b5-8 and the 'safer' hypothesis, the 'subtler' answers at 105b8-c2 come together. The subtler answer tracks why the soul makes a body alive, and the original safe answer tracks why the Form Life makes the soul alive.<sup>4</sup> Analogously, the subtler answer tracks why the unit makes an ἀριθμός odd, and the original safe answer (presumably) tracks why the Form Odd makes the unit odd. What my analysis shows is that Plato's argument for the immortality of the soul depends on the Greek mathematical understanding of number, and moreover, if I am right, makes room for what Aristotle called τὰ μαθηματικά, and attributed them to Plato, who supposedly held these objects to be τὰ μεταχύ, between Forms and sensible particulars.5

I first discuss the passage at 103e9 to 104b4, where Socrates demonstrates an expansion of his initial hypothesis given at 100d4-7. The initial hypothesis is that all F things are F by virtue of participating in the F. In the expansion, Socrates claims that there is something else that is not the F, but nevertheless is called F by virtue of its character. (103e2-5) Socrates demonstrates the claim with numbers, separating the odd from the even. Yet the two series of number do not completely line up. In the Greek, the odd numbers are in the feminine singular while the even numbers are in the neuter plural. I examine these two series of number, and argue that the differences between these two demonstrate essential and non-essential characteristics, aligning with two ways of being for number. These differences are important for understanding the analogy in the final proof, that three is odd as soul is alive, and their two ways of being, itself by itself and with extended objects. Then I examine the passage in the Phaedo at 105c6, where Socrates discusses the subtler answers, and present interpretations from other commentators to show that how the term μονάς is understood makes the difference for evaluation of the final proof. I provide my own translations, however I encourage the reader to compare several different translations. Here I discuss μονάς, ἀριθμός, περιττός and ἄρτιος within the context of the passage, and then go on to examine the logic of the final proof. I argue that Plato seamlessly changes from a collection of three things ( $\tau \dot{\alpha} \tau \rho i \alpha$ ) to the character of being three ( $\dot{\eta}$  τρίας), that these arithmoi are collections of things in the first case and a collection of Form characters, or 'units' in the second case. Scholars tend to take  $\dot{\eta}$  τρίας as a Form, and posit Form numbers for Plato. However, this is a mistake. If three is an arithmos, it cannot be a Form. There are no Form numbers, when numbers are understood to be a limited multitude.<sup>6</sup>

# FIRST PASSAGE

In the first passage under discussion, Socrates distinguishes the Forms Odd and Even from their instantiations. The first line of numbers is odd 'by nature', the second line of numbers is even but Socrates leaves out the 'by nature' qualification.<sup>7</sup> The other distinction he makes that few commentators point out is that the odd line of numbers is in the feminine singular whereas the even line of numbers is in the neuter plural. This is from 103e9 to 104b4:

> [Τ1] ἦρα μόνον τῶν ὄντων τοῦτο γὰρ ἐρωτῶ — ἢ καὶ ἄλλο τι ὃ ἔ στι μέν ούχ ὅπερ τὸ περιττόν, ὅμως δὲ δεῖ αὐτὸ μετὰ τοῦ ἑαυτοῦ ὀνόματος κ αὶ τοῦτο καλεῖν ἀεὶ διὰ τὸ οὕτω πεφυκ έναι ώστε τοῦ περιττοῦ μηδέποτε ἀπο λείπεσθαι; λέγω δὲ αὐτὸ εἶναι οἶον κα ι ή τριὰς πέπονθε και ἄλλα πολλά. σκ όπει δὲ περὶ τῆς τριάδος. ἆρα οὐ δοκε ῖ σοι τῷ τε αὑτῆς ὀνόματι ἀεὶ προσαγο ρευτέα εἶναι καὶ τῷ τοῦ περιττοῦ, ὄντ ος ούχ ὅπερ τῆς τριάδος; ἀλλ' ὅμως ο ύτως πέφυκε καὶ ἡ τριὰς καὶ ἡ πεμπτὰ ς καὶ ὁ ἥμισυς τοῦ ἀριθμοῦ ἅπας, ὥστε ούκ ὢν ὅπερ τὸ περιττὸν ἀεὶ ἕκαστος α ύτῶν ἐστι περιττός καὶ αὖ τὰ δύο καὶ τὰ τέτταρα καὶ ἅπας ὁ ἕτερος αὖ στίχος τοῦ ἀριθμοῦ οὐκ ὢν ὅπερ τὸ ἄρτιον ὅμως ἕκαστος αὐτῶν ἄρτιός ἐστιν ἀ εί· συγχωρεῖς ἢ οὔ;

> Is it the case then that there are only these beings — for this is the question you must answer — or rather is there something else which is not the Odd, but all the same it is necessarily with its own name and this always is called odd on account of its nature, with the result that it is never separate from the odd? I mean the triad to be in this way and the many others. Consider the triad. Does it seem to you to always be with its own name and to be odd, not just being

three? And so it is this way by nature too the triad and the quintet and the whole half of the multitude, with the result that none are the Odd, but each of them is always odd; and again the twos and the fours and the whole other line of number, none is the Even, nevertheless each of them always is even. Are you in agreement, or not?

There is much disagreement in the literature with respect to these two lines of number. Some take it for granted that Socrates is talking about the same line of number, as if he were simply talking about the cardinal or natural number system: 0, 1, 2, 3, 4, 5, etc. That assumption, however, doesn't acknowledge the differences that Socrates makes not just with the odd and even numbers, but the number in kind. Also, our cardinal numbers begin with 0, which at the time of Plato the Greeks didn't have.8 Many assumed in the Phaedo that ή τρίας (104a4, 104a8, 104e5, 104e8), τρία (104c3), τρισίν (104d6) and τὰ τρία (104c1, 104e1, 104e3, 106a1, 106b5, 106c5) are the same in that they all mean 'three' and are just simply, 'three'.9 While it is true that these terms all designate 'three,' it is quite possible that when Plato writes  $\dot{\eta}$  τρίας, he either means the Form three or a triad (an ontological difference to be sure), accessible to the mind, and that when he writes τὰ τρία he means a trio of bodied particulars, what we see in the sensible world.<sup>10</sup> In fact, these two kinds of number line up with other passages in Plato's dialogues, Republic VI 510e5-511a2, VII 525a7-c6, Theaetetus, 198c1-2, Philebus 56d4-57a4, where number comes in kinds and for different purposes. So when the Greek text in the Phaedo at 104a-106c5 switches in gender and number for the number three, for example, we should not assume, as many

have, that these numbers share the same ontological status.<sup>11</sup> Instead, we need to ask what these changes mean for the argument that contains them.

I suggest that the passage under consideration, 103d9-104b7, uses not one but two different lines of number, if we were to take the differences in number and gender seriously. The objects Socrates discusses, what we call odd, τὸ περιττόν, are not the Form, yet he points out that we always call them odd on account of their nature,  $\delta$ ià tò oὕτω πεφυκέναι, because the odd never leaves them. Such is the nature of the triad, quintet, and the half of the whole multitude; ή τριὰς, ή πεμπτὰς καὶ ὁ ἥμισυς τοῦ ἀριθμοῦ ἅπας, 104a7-b1. As mentioned before, this half is in the feminine singular. Ronna Burger points out that this line of number is odd by nature and the 'by nature' designation is explicitly made for the odd numbers, whereas when Socrates discusses the other line of number, the even,  $\tau \dot{o}$ ἄρτιον, the "by nature" designation is explicitly left out.<sup>12</sup> Admittedly, there is much we don't know about these two lines of number. The odd numbers he lists here may not be inclusive of every number, or they may be. The same is true for the even numbers he lists. What we do know, though, is that the odd numbers he lists are odd 'by nature' and that these are designated in the feminine singular. In contrast, the even numbers that he lists are not even by nature, or at least he doesn't say that they are, and these even numbers are designated in the neuter plural. Burger has argued that the reason the even numbers are not by nature even is because being even is simply that which is capable of being divided into two equal parts, and this is common to not just numbers but also infinitely divisible magnitudes.<sup>13</sup> What her account entails then is that only discrete and countable numbers

containing units can be odd. Burger assumes that Socrates is talking about the same number line and doesn't account for the differences in number (i.e. singular and plural) of the two lines. Moreover, Burger's account leaves out the first disjunct of Euclid's definition of odd in Book VII.7, that in contrast to an even number, the odd can also be that which is not divisible into two equal parts. What is more probable and what I offer here is that the 'by nature' designation for  $\pi\epsilon\rho\iota\tau\tau \delta\varsigma$  in the first line of ἀριθμός means to establish a necessary relation with an object and its essential characteristic, and that the leaving out 'by nature' may mean an accidental relation, not necessary to the objects themselves.14

If this is right, then Socrates is not distinguishing the nature of odd and the even, two aspects of arithmos, rather, Socrates is distinguishing the nature of two kinds of number: whether they are themselves by themselves or whether they are connected to a composite, to a body. These τὰ τρία are odd not by nature, in contrast to the cases of  $\dot{\eta}$  τριάς,  $\dot{\eta}$  πεμπτάς καί ό ἥμισυς τοῦ ἀριθμοῦ ἅπας, at 104a7-b1, which are odd by nature. We call three objects odd, as with the neuter plural even numbers listed at 104b2-3, but these composite objects are not their number by nature, but by the number they happen to have. I take Burger's point that the first line of number at 104a7-8 is odd by nature, but not because of the nature of odd numbers, but because of the nature of abstract numbers, numbers 'themselves by themselves,' separate from their sensible objects. My view leaves open the possibility, in contrast to Burger's interpretation, that even numbers are also even by nature if they are even in the abstract. Magnitudes and counted objects, which are necessarily composite, are not odd or even by their nature. In other words, when you have a solid or when you have a bundle of objects, you only discover an odd or an even number when you further limit the multitudes into those which can be equally divided and those that cannot.

## SECOND PASSAGE

Socrates prepares for the analogy of  $\mu ov\dot{\alpha}\varsigma$ and  $\psi v\chi\dot{\eta}$  in our second passage at 105b8-c6. Note that this passage is where Socrates transitions from the safe hypothesis to the more subtle hypothesis posited here.<sup>15</sup> This subtler hypothesis claims that there is something else in addition to the *F* that generates characteristics in objects:<sup>16</sup>

> [T2] εἰ γὰρ ἔροιό με ῷ ἂν τί ἐν τῷ σώματι ἐγγένηται θερμὸν ἔσται, οὐ τὴν ἀσφαλῆ σοι ἐρῶ ἀπόκρισιν ἐκείνην τὴν ἀμαθῆ, ὅτι ῷ ἂν θερμότης, ἀλλὰ κομψοτέραν ἐκ τῶν νῦν, ὅτι ῷ ἂν πῦρ· οὐ δὲ ἂν ἔρῃ ῷ ἂν σώματι τί ἐγγένηται νοσήσει, οὐκ ἐρῶ ὅτι ῷ ἂν νόσος, ἀλλ' ῷ ἂν πυρετός οὐδ' ῷ ἂν ἀριθμῷ τί ἐγγένηται περιττὸς ἔσται, οὐκ ἐρῶ ῷ ἂν περιττότης, ἀλλ' ῷ ἂν μονάς, καὶ τἇλλα οὕτως. ἀλλ' ὅρα εἰ ἤδη ἰκανῶς οἶσθ' ὅτι βούλομαι.

> So, if you were to ask me by whichever thing in a body would generate heat, I would not give the safe answer that is unlearned, that would be heat, but a more sophisticated answer from those now, fire; nor if you were to ask me by whichever thing in a body generates illness, I would not answer illness, but rather fever; nor if you were to ask me by whichever thing would generate odd in an *arithmos*, I would not answer that of oddness, but a unit, and the others, too, in this way. But see whether you already grasp sufficiently what I want you to know.

In T2, Socrates gives three examples that generate characteristics in things, and these go beyond the safe hypothesis given at 100c-101c.<sup>17</sup> The first two examples should be distinguished from the third, as the first two are about affects in bodies, and the third is about affect in an arithmos, which may or may not be bodied. Moreover, when Socrates makes the final steps in his last proof for the immortality of the soul, he doesn't use fire, fever or even heat, for that matter, but the Forms Odd and Even and the numbers three and two. (105c9-e9) In any case, the first two examples use fire and fever to generate heat and illness in bodies, and note that bodies are extended objects.<sup>18</sup> The third example is about the generation of an affect in an  $\dot{\alpha}\rho_i\theta_\mu\dot{o}\varsigma$ , which need not be an extended object. While the point seems obvious, it would be beneficial to try to understand Plato's choice of *arithmos* here within the context of Greek mathematics.

Arithmos is what we tend to translate as 'number', but the concept of Greek number is much different from our own, the proper definition is a limited multitude. (Euclid VII.2) Plato's use here is that the  $\mu ov \dot{\alpha} \varsigma$  generates  $\pi \epsilon \rho i \tau \tau \dot{\delta} \varsigma$ , the "odd" in a multitude. Recall from before that the definition we get from Euclid for περιττός comes in a disjunction, the 'odd' is that which cannot be divisible in two equal parts, [or] that which differs from an even number by a unit. (Euclid VII.7) Note that the first disjunct includes magnitudes or extended objects such as lines, planes and bodies, as well as discrete units but the second disjunct could only apply to discrete comparable units. It is important for us to realize that arithmos can be any collection of units, or a collection of sensible things depending on the context, but what arithmos cannot be is a Form.<sup>19</sup> Why arithmos cannot be a Form is that, by definition, arithmos is a 'many' and so cannot be 'one'. While μονάς is said to be 'one' and follows the first definition in Euclid VII, *monas* is that by which each thing is called 'one', different interpretations have led to different understandings of T2, which lead to more or less negative evaluations of the final proof. Part of the interpretive issue in T2 is how commentators have understood μονάς within the context of the passage.

In Greek, μονάς could be understood as 'unity,' 'oneness,' 'one,' or a 'unit.' Often these terms are used interchangeably. I defend here the 'unit' in T2. I'll briefly go through the first three possibilities before I give my own account. Mováç understood as 'unity,' implies that the whole is a singularity, a 'one' without parts (Philebus 15b1-8, Parmenides 137c6-d3), in other words, the totality of one. Yet 'unity' in our parlance, implies parts, just like a whole implies parts. Not only does unity without parts sound like a contradiction, it does not get us any closer to understanding Plato's choice of μονάς in T2. 'Oneness,' is that aspect of being 'one', which can also be another word for τὸ ἕν, the Form One. But it is not simply one that is under consideration here, but the unit and its relation to a multitude. 'One' for the Greeks is that beginning from which we count, but it is not something to be counted (Laws 818c4-6). While we think of 'one' as the first natural number, it cannot be stated too often that 'one' is not an ἀριθμός for Plato, as it is not a multitude.<sup>20</sup> Ἀριθμός, which is a plurality, is that which is τὰ μεταχὺ, 'between' one and the unlimited. (Philebus 16c10-e2) The reason why we should take μονάς to mean 'unit' at 105c6 is due to the necessary connection that the single unit has to περιττός after a plurality of units have been divided into two parts. When the two parts are equal to each other, the whole ἀριθμός is said to be ἄρτιος, 'even'. (Laws 895e1-8; Euclid VII.6) When the two unequal parts differ by a single unit, the whole ἀριθμός is said to be περιττός. (Euclid VII. 7)

Remember that any multitude can be a kind of ἀριθμός, and can be divided equally or unequally. We already saw two kinds of arithmos in our T1 passage, distinguishing abstract odd arithmos from embodied even arithmos. Likewise, there are unit, plane, and solid arithmoi. In the Euthyphro, Plato uses for example an isosceles arithmos, what we call an 'isosceles triangle'. Plato says there at 12d8-10 that the isosceles is also ἄρτιος, which must mean that an isosceles triangle can be divided equally, whereas a scalene triangle cannot. In the Theaetetus at 148a6-9, where Plato introduces to us the problem of incommensurables, ού συμμέτρους, he has Theaetetus discuss square and oblong plane άριθμοί — these are multitudes of units arranged in the shape of rectangles, formed by the product of like numbers or unlike numbers.<sup>21</sup> While περιττός and ἄρτιος can be predicated to any object, to any multitude, it is only the lone  $\mu ov \dot{\alpha} \varsigma$  that is left over from what can be equally divided as 'discrete and indivisible units' that marks  $\pi \epsilon \rho \iota \tau \tau \delta \varsigma$  in that kind of ἀριθμός.<sup>22</sup> Though μονάς itself is not a number, it is still  $\pi\epsilon\rho\iota\tau\tau \circ \varsigma$ .<sup>23</sup> Why this would be true for the Greek mathematicians can be found in Euclid IX, proposition 27.24 Traditionally μονάς has not been understood quite this way in the passage under consideration.

### Movάς Interpretations in T1

There are scholars who try to understand *monas* within the confines of the arguments in the *Phaedo* without attention to the Greek mathematical understanding of *monas* and *arithmos*. Burnet says that *monas* means here

'unity' but then he finds fault with Plato's argument. In a note he says that 'there are other odd numbers than the number one'.<sup>25</sup> Burnet takes 'unity' to mean 'one', a common understanding of the term. Yet as noted before, 'one' could not be an arithmos for Plato, since one is not a multitude. One might think that an advantage to Burnet's account is that if 'unity' were identified with 'one' then monas would be sufficient to generate the odd in arithmos. Yet Plato is not looking for a sufficient generation, since at 105d1-3 he will need to use an exclusion of a specific opposite for the final proof, something that will necessarily exclude death from the soul, just as he needs something that will necessarily exclude the even in a particular arithmos. The crucial turn in the final proof is at 105d6-12, where soul necessarily excludes death. So whatever has soul is not dead. But to prove that the soul is immortal, ἀθάνατος, Socrates must show that soul is the kind of thing that exists separately from the body, and never admits death. As Kanayama points out, άθάνατος does not simply mean 'alive' it means not admitting death.<sup>26</sup> Whether it is the case that fire and fever only apply to extended objects or something else, Socrates leaves these subtle answers, and focuses on three being odd by the unit in the final proof. Yet there are more ways commentators have thought about μονάς in the T2 passage.

Bluck says that μονάς is 'oneness' but leaves 'oneness' out of his analysis of the final proof.<sup>27</sup> 'Oneness' might work for a generation of περιττός if we understand περιττός as that which cannot be divided into two equal parts. We'd have to understand 'oneness' here as something that cannot be divided at all for it to be in harmony with the account of the odd in Euclid VII.7. Yet, 'oneness' alone would not generate περιττός in an ἀριθμός since an *arithmos* is more than one and so can be divided. In other words, 'oneness' could not be applied to a multitude. Nor can μονάς be understood as a Form, as Hackforth did, though interestingly in his text he translates μονάς as 'unit,' and explains the analogy in this way: 'Just as μονάς brings up περιττότης and excludes ἀρτιότης, so ψυχή brings up ζωή and excludes θάνατος. All these are Forms.'28 It doesn't make sense for a unit to be a Form, as there are multiple units in an  $d\rho_i\theta_\mu \delta\varsigma$ , and if what Plato says about forms at 78d1-d7 holds, then a Form can only be μονοειδές ὂν αὐτὸ καθ'αὑτό, as Bluck translates, 'being of single Form when taken by itself,<sup>29</sup> — there can only be one of its kind, not many. Yet I agree with Hackforth when he says that µováç excludes ἄρτιος by being the unit 'left over' in the middle.<sup>30</sup> This, in fact, is closer to the second disjunct of the definition of odd in Euclid's Elements VII.7.

There are others who completely leave out monas in their evaluation of the final proof and not surprisingly their evaluations often claim that the final proof fails.<sup>31</sup> Bostock leaves out a discussion of μονάς altogether in his analysis of the analogy of soul being alive and three being odd, and he claims that three must be a Form.<sup>32</sup> This leads to interpretive grief for Bostock, as he says the more subtle causes are 'a mixed lot': some being Forms, others being Forms-in--somethings and others being physical stuffs. Bostock says that they give us little guidance as to how to understand the most important cause, soul.<sup>33</sup> Schiller, though he suggests intermediates, leaves out a discussion of μονάς.34 Yet it is precisely  $\mu ov \dot{\alpha} \varsigma$ , as itself by itself, and as a collection of equally divided units that give us the designations τὸ ἄρτιον and τὸ περιττόν. As demonstrated previously in the T1 passage, the multitude here can mean sensible or non--sensible objects, whatever can be 'counted', whether that is through our senses or through

thought, whether what are counted come from extended or non extended objects.<sup>35</sup>

To recap, for the Greeks and for Plato, most importantly, number, i.e., arithmos, is a limited multitude — it is what can be counted. We see evidence for this in the Theatetus at 198c4-6, where Theatetus agrees with Socrates that 'we should take counting to be nothing other than seeing how many (posos) any number happens to be': τὸ δὲ ἀριθμεῖν οὐκ ἄλλο τιθήσομεν τοῦ σκοπεῖσθαι πόσος τις ἀριθμὸς τυγχάνει ὤν. So one as such is not a number because there is not a multitude in one, but simply, one unit, the monad,  $\dot{\eta}$  µováς. The unit is by means of which we count, but it is not what is counted. On this view, two begins the number series. We see 'two' beginning the number series in the T1 passage at 103e9 to 104b4. Every number is not just a limited multitude, but every countable number also contains comparable units equal to themselves.<sup>36</sup> Units are τὰ ἴσα 'equals' to one another in a multitude. Moreover, we should remember that Plato avails himself of more than one kind of number. In his Republic at 525 b-d Plato distinguishes pure number from counting things that you can see and touch. The passages in Plato's Euthyphro, Theaetetus, and Philebus previously discussed demonstrate that μονάδα, 'units' are counted, but the kind of objects counted, determine the relations among units, and these relations are determined by Forms. For now, what is important to conclude about passage 105c4-6 in the Phaedo is that Plato has a very specific relation of μονάδα and their ἀριθμός and the Forms περιττός and ἄρτιος in mind for the final proof.

# THIRD PASSAGE

The third and last passage under consideration is the argument by analogy in the final proof (105c9-105e8):

[T3] Ἀποκρίνου δή, ἦ δ' ὅς, ῷ̃ ἂν τί ἐγγένηται σώματι ζῶν ἔσται; Ωι ἂν ψυχή, ἔφη. Ούκοῦν ἀεὶ τοῦτο οὕτως ἔχει; Πῶς γὰρ οὐχί; ἦ δ' ὅς. Ψυχὴ ἄρα ὅτι ἂν αὐτὴ κατάσχῃ, ἀεὶ ἥκει ἐπ' ἐκεῖνο φέρουσα ζωήν; Ήκει μέντοι, ἔφη. Πότερον δ' ἔστι τι ζωῆ ἐναντίον ἢ οὐδέν: Έστιν, ἔφη. Τί; Θάνατος. Ούκοῦν ψυχὴ τὸ ἐναντίον ῷ αὐτὴ ἐπιφέρει ἀεὶ οὐ μή ποτε δέξηται, ὡς ἐκ τῶν π ρόσθεν ώμολόγηται; Καὶ μάλα σφόδρα, ἔφη ὁ Κέβης. Τί οὖν; τὸ μὴ δεχόμενον τὴν τοῦ ἀρτίου ίδέαν τί νυνδή ώνομάζομεν; Άνάρτιον, ἔφη. Τὸ δὲ δίκαιον μὴ δεχόμενον καὶ ὃ ἂν μουσικόν μή δέχηται; Άμουσον, ἔφη, τὸ δὲ ἄδικον. Εἶεν· ὃ δ' ἂν θάνατον μὴ δέχηται τί καλοῦμεν; Άθάνατον, ἔφη. Ούκοῦν ψυχὴ οὐ δέχεται θάνατον; Οű. Άθάνατον ἄρα ψυχή. Άθάνατον. Εἶεν, ἔφη· τοῦτο μὲν δὴ ἀποδεδεῖχθαι φῶμεν; ἢ πῶς δοκεῖ; Καὶ μάλα γε ἱκανῶς, ὦ Σώκρατες.

Then tell me, what in a body will generate life? The soul, he said. Does it always do this? Why wouldn't it? Then isn't it soul that always brings life upon that which it occupies? Indeed it brings. Then is there something opposite to life, or not? There is. What? Death. Then isn't the opposite to which soul brings never admitted, as we agreed before? Indeed, most definitely said Cebes. What then? What name did we call just now the Form that does not admit the even? Uneven, he said. And what do we call that which doesn't admit justice and the musical? Un-musical and un-just. It is: what do we call that which wouldn't admit death? The un-dead. Then isn't it the soul that doesn't admit death? Yes. Then the soul is un-dead. It is, he said; Would you say that we proved this, or how does it seem to you? Indeed, sufficiently proved, Socrates.

Soul is that 'whatever thing' that generates life in a body. This answer matches the  $\mu ov \dot{\alpha} \zeta$  that generates odd in an *arithmos*. As noted previously,  $\psi v \chi \dot{\eta}$  and  $\mu ov \dot{\alpha} \zeta$  are decidedly different from fire and fever, the other more subtle answers. It is not only that  $\mu ov \dot{\alpha} \zeta$ and  $\psi v \chi \dot{\eta}$  necessarily and sufficiently generate their essential characteristics, being odd and being alive, it is that what ἀριθμός and  $\psi v \chi \eta$  are here is ambiguous; they have a double existence. Number can exist abstractly, and necessarily be odd or even by nature, such as περιττός in ή τριὰς, καὶ ή πεμπτὰς and all the rest. (104a8) These are not sensible bodies counted, but rather, collections of units themselves. Or, number can be called odd or even whenever it happens to exist in bodies, such as the ἄρτιος in τὰ δύο καὶ τὰ τέτταρα and all the rest. (104b2-3) These are the sensible bodies that happen to be numbered, there is nothing 'by nature' that makes them their number. Sensible bodies have number; they are not identified as their number. This double existence matches that of  $\psi v \chi \dot{\eta}$ . For soul exists itself by itself, a sort of abstraction or separation from the body, and soul can exist in a body. Notice that bodies are not essentially connected to soul, any more than bodies are essentially connected to the equals themselves.<sup>37</sup> However, the unit is essentially connected with its Form Odd. Movάς, itself by itself, carries with it the Form character Odd and so makes a collection of equals themselves 'odd' whenever a unit happens to be left over from them being equally divided. Likewise, ψυχή, itself by itself, always carries with it the Form characteristic Life and so 'enlivens' whatever body it is in. Thus soul is essentially connected with its Form Life. This is the force of line 105d2-3:

Ψυχὴ ἄρα ὅτι ἂν αὐτὴ κατάσχηι, ἀεὶ ῆκει ἐπ'ἐκεῖνο φέρουσα ζωήν;

Therefore, whatever the soul occupies, isn't it always bringing life to it?

Soul always has and carries with it life, and so it follows by nature that soul is always with life. While it is clear at 105d2-3 that the subject of κατάσχηι is ψυχή, so, 'occupies' or 'dwells in' – κατάσχηι here is used in a double sense.<sup>38</sup> Whenever the soul occupies, it always carries with it life. But it follows too that soul is occupied by that which it carries. Not only is the soul compelled by what it carries, in a certain sense, the opposite of what it carries affects the soul and the body that soul occupies. What is true for the soul is true for whatever body it occupies, but only when it occupies it. Right away we should recall 104d1-3:

> ἆρ' οὖν, ἔφη, ὦ Κέβης, τάδε εἴη ἄν, ἃ ὅτι ἂν κατάσχῃ μὴ μόνον ἀναγκάζει τὴν αὑτοῦ ἰδέαν αὐτὸ ἴσχειν, ἀλλὰ καὶ ἐναντίου αὐτῷ ἀεί τινος;

> But then Cebes, he said, wouldn't they be those things which compel whatever they occupy to contain not only its own Form but also always the Form of some contrary?

The principle at 104d1-3 is recalled at 105d2-3. Together, they bring the logical force of the final proof to its conclusion. Notice that Socrates moves from the opposite of the soul's essential characteristic life (death) to the opposite of the essential characteristic of being odd, at 105d13-15 with an unstated premise in the proof. I suggest that the unstated premise follows that of 105d2-3:

> Μονάς ἄρα ὅτι ἄν αὐτὴ κατάσχηι, ἀεὶ ἥκει ἐπ'ἐκεῖνο φέρουσα περιττὸν;

> Therefore, whatever the unit occupies, isn't it always bringing the odd to it?

If we agree that this premise is suppressed in the proof, then we can track the logical connection that Socrates makes, from the soul being the generative cause of life in a body (105c9), and the exclusion of the opposite of soul's essential characteristic which gives soul its immortal status (105e7). Similarly, the lone unit is the generative cause of a multitude of equals being odd, i.e., the character of being odd (105d13) and the exclusion of their opposite, equal multitudes being equally divided, that gives three objects their uneven status. Now we can put together the three texts we've discussed to examine the final proof in the *Phaedo*.

Soul, itself by itself, is compelled by the life-giving characteristic and necessarily brings that characteristic to a body, making that body alive. So, too, the trio of units are compelled by that odd-giving characteristic, the left over unit,  $\mu o \nu \dot{\alpha} \varsigma$ , and they necessarily bring this characteristic to a trio of bodies, making those bodies that happen to be a trio, odd. The opposite of life is death, just as the opposite of odd is even. While it is true that the soul itself can never be dead (104d1-3) because it will always exclude the opposite character to which it necessarily carries, it does not follow, as Strato and much later Keyt and others following him would argue, that the embodied soul will never die.39 For ψυχή is not always in a body, just as sensible objects do not always keep their number. This is the force of the ontological distinction of non--sensible intermediates on the one hand and sensible bodies on the other.

Though the participants agree that they've proved the soul's immortality (105e9) Socrates continues, for he needs to keep his promise to Cebes (88b, 95b-e) that the soul be  $\dot{\alpha}\nu\dot{\omega}\lambda\epsilon\theta\rho\sigma\nu$ , indestructible, as well as  $\dot{\alpha}\theta\dot{\alpha}\alpha\tau\sigma\nu$ , immortal.<sup>40</sup> As Burnet's note tells us, we still have two possible alternatives. Even though the soul will not admit death, Socrates still needs to show that the soul will 'withdraw' (the first alternative) and not perish with the body (the second alternative). The case of τὸ ἀθάνατον is, Burnet says, *ipso facto* ἀνώλεθρον.<sup>41</sup> This is where many commentators find fault with the final proof.<sup>42</sup>

Bostock offers a reconstruction of the argument to demonstrate that Socrates is question begging:

1. If there is anything that is indestructible, then what is immortal is indestructible (d2-4).

2. But there is something indestructible, namely God and the Form of life (d5-7).

Therefore: What is immortal is indestructible.<sup>43</sup>

Bostock argues that the premises presume the conclusion of what they are trying to prove, and he says we have no reason to accept the first premise anyway. Yet we can do better than Bostock, as 'one man's begging the question might be another man's tacit assumption.'<sup>44</sup> There is a way to unpack the hidden premises in this very last stage of the argument. Recall what has already been established in the first part of the final proof:

- 1. When objects lose or gain characteristics, they undergo change. (103b2--e1)
- 2. When objects lose their essential characteristics, they cease to exist. (103e2--103e5)
- 3. There are some objects that never lose their essential characteristics. (103e6--104b1)
- 4. ή τριάς and ή ψυχή never lose their essential characteristics, 'odd' and 'life'.
  (104a3-8, 105d2-3)

- What never loses its essential characteristic will always exclude that essential characteristic's opposite from coming into being in that object. (104c7-d3)
- 6. The opposite of odd is even, opposite of life is death. (104d12-14, 105d6-9)

From premises 1-6, we can conclude that  $\dot{\eta}$  τριάς will never be even, and soul will never be dead. That is what the first part of the final proof establishes, formally. Now, from the conclusion that soul will never be dead, and three will never be even, we get the following:

- 7. Whatever never loses its essential characteristic is everlasting. (106a1)
- 8. Whatever loses its essential characteristic is not everlasting. (106b3-c8)
- 9. Soul never loses its essential characteristic life. (105d3-e4)
- 10. Whenever a living body loses its soul, it dies. (106e3)
- Whatever has as its essential characteristic, life, is immortal. (premises 1-6)
- 12. Whatever is immortal is everlasting. (premises 7-11)

Therefore: A living body is neither immortal nor everlasting (106e5). Soul, itself by itself, is immortal and everlasting (106e9-107a).

So snow has its essential characteristic cold, but it can lose this characteristic and it will no longer be snow. Socrates may lose unessential characteristics and still remain who he is: whether Socrates is tall or short, he is Socrates. Yet Socrates as a composite body and soul is not everlasting because the composite is an extended, sensible object. Moreover, being alive is not an essential characteristic to a body, any more than a left over monad is an essential characteristic to a multitude of units. Although life is an essential characteristic for Socrates being alive, his composite loses this characteristic at death. Three,  $\dot{\eta} \tau \rho \iota \dot{\alpha} \varsigma$ , on the other hand, will always have the left over unit in its multitude of equally divided units and so will always be odd. Likewise, soul will always carry life and so will never die, soul will always be immortal. Therefore soul and three, since they are objects that will always bear their essential characteristics, are everlasting.

To demonstrate this last phase of the argument, it is instructive that Socrates begins with ἀριθμός, specifically, the neuter plural τὰ τρία (106a) and not the feminine singular ή τριάς (104e8).45 Socrates is using an embodied trio, so, sensible particulars that happen to be three and so odd, and not the abstract trio separated from bodies for this part of his argument. The embodied trio is not necessarily three, for at any time another bodied unit could come along or be taken away and the περιττός would withdraw. So the triad of bodies are only temporarily odd and so never everlasting: if the bodies themselves were destroyed, then the trio would withdraw. To speculate, Plato doesn't use ή τριάς here because it is not the three itself that he needs for the argument (for τριάς is by nature odd); he needs to start with the embodied three, just like he needs to start with the embodied soul, to convince Cebes and Simmias that the soul of Socrates is indestructible as well as immortal once it separates from the body.

# CONCLUSION

I have analyzed some of Plato's passages in the *Phaedo* with careful attention to  $\mu ov \dot{\alpha} \varsigma$  and its analogy with  $\psi v \chi \dot{\eta}$ , and how they logically connect the propositions in the final proof to the conclusion that the soul is immortal, and since it is immortal, soul is everlasting. While other commentators point out the logical flaws and inconsistencies in the arguments, I showed that Plato avails himself two ontological distinctions of number: as embodied in sensible particulars and as abstract collection of units. In a similar fashion, we should understand soul, like the unit, to share this dual status, in that both can become embodied and joined with sensible objects and both can be understood as existing separately from bodied particulars. Yet souls and units, although understood as responsible for generating characteristics in objects, are not Forms themselves, but bearers of Form characteristics, for they are able to effect change in bodies, yet unlike sensible objects, they never lose their essential characteristics. Perhaps this dual role for souls and units is due to their ontological status, τὰ μεταχύ, between Forms and sensible particulars.

# REFERENCES

- Altman, W. 2016: *The Guardians on Trial*, Maryland: Lexington Books 2016.
- Annas, J. 1975: "On the 'Intermediates,"*Archiv für Geschichte der Philosophie* 57 (1975), 146-166.
- Archer-Hind, R. D. 1883: *The Phaedo of Plato*, London: Macmillan and Co. 1883.
- Bluck, R. S. 1955: *Plato's Phaedo*, London: Routledge & Kegan Paul Limited 1955.
- Bostock, D. 1989: *Plato's Phaedo*, Oxford: Clarendon Press 1883.
- Burger 1984: *The* Phaedo: *A Platonic Labyrinth*, New Haven: Yale University Press 1984.
- Burnet, J. 1892, 2018: *Early Greek Philosophy*, London and Edinburgh: Adam and Charles Black. Kessinger Publishing's Reprints. 1892, 2018.
- —. 1900, 1901: Platonis opera. Vols, 1 and 2, Oxford: Clarendon Press, Thesaurus Linguae Graecae\* Digital Library 1900, 1901.
- Cherniss, H. 1962: *The Riddle of the Early Academy*, New York: Russell & Russell 1962.

- Gallop, D. 1975: *Phaedo/Plato*, Translated with notes by David Gallop, Oxford: Clarendon Press 1975.
- —. 1993: Phaedo, Oxford World Classics Paperbacks, Oxford: Oxford University Press 1993.
- Euclid. 2010: *Euclid's Elements*. Tr. Thomas Heath. Chelsea: Green Lion Press 2010.
- Hackforth, R. 1995: *Plato's Phaedo*, Cambridge: Cambridge U. Press 1995.
- Heath, T. 1981: *A History of Greek Mathematics*, New York: Dover Publications 1981.
- ----. 1931: Manual of Greek Mathematics, Oxford: Clarendon Press 1931.
- Kanayama, Y. 2000: 'The Methodology of the Second Voyage and the Proof of the Soul's Indestructibility in Plato's Phaedo.' Oxford Studies in Ancient Philosophy. 18 (2000), 41-100.
- Keyt, D. 1963: 'The Fallacies in "Phaedo" 102a-107b' *Phronesis*, 8 (1963), 167-172.
- Klein, J. 1968: Greek Mathematical Thought and the Origin of Algebra, tr. Eva Brann, Cambridge: The M.I.T. Press 1968.
- Lasserre, F. 1966: *Birth of Mathematics in the Age of Plato*, London: Hutchenson of London 1966.
- O'Brien, D. 1967: 'The Last Argument of Plato's Phaedo I,' *The Classical Quarterly* 17 (1967) 168-231.
- Pakaluk, M. 2003: 'Degrees of Separation in the *Phaedo.*' *Phronesis*, 48 (2) 2003, 89-113.
- Plato. 1993: *Plato Phaedo*, ed. Rowe, C. Cambridge University Press 1993.
- —. 1997: 'Phaedo,' tr. G.M.A. Grube, *Plato Complete Works*, ed. J. Cooper. Indianapolis: Hackett Publishing 1997.
- —. 1967: Phaedo, in ed. J. Burnet. Platonis opera, Vol. 1, Oxford: Clarendon Press 1967.
- Pritchard, P. 1995: *Plato's Philosophy of Mathematics*, Sankt Augustin: Academia Verlag 1995.
- Prince, B. 2011: 'The Form of Soul in the *Phaedo*,' PLATO, 11 (2011).
- Rist, J.M. 1964: 'Equals and Intermediates in Plato's Phaedo,' *Phronesis* 9 (1964), 27-37.
- Ross, W.D. 1924, repr. 1970 [of 1953 corr.edn.]: Aristotle's Metaphysics, 2 vols., Oxford: Clarendon Press in Thesaurus Linguae Graecae<sup>\*</sup> Digital Library 1924.
- Ross, W.D. 1963: *Plato's Theory of Ideas*, Oxford: Clarendon Press 1963.
- Schiller, J. 1967: 'Phaedo: 104-105: Is the Soul a Form?' *Phronesis* 12, (1967) 50-58.

#### Stone, S. 2014: 'The role of arithmos in Plato's Phaedo,' Southwest Philosophical Review, 30 (2014), 137-149.

# NOTES

1 Rist 1964, 30.

2 Aristotle *Metaphysics A*. 992b16; Annas 1975, 148, 155, 162.

One might take issue with my use of Euclid's 3 Elements, a text composed later than Plato in Alexandria, to justify an interpretation of arithmetical objects in Plato's dialogues. Klein 1968, 43 conjectures that Book X, which addresses incommensurables, which also presupposes content from Books VII, VIII and IX, comes from the ideas of Theaetetus. It is evident that Plato was familiar with the work of Theaetetus as shown in the dialogue named after him. Lasserre 1964, 16-17, tells us that we learn from Proclus that there were several texts called 'Elements' before Euclid's, and that Euclid had incorporated many of the mathematical principles from the prior texts. This suggests that much of the content in Euclid's Elements was known to the Greek mathematicians of the 4th century, BCE, and perhaps even earlier. It is no coincidence that Plato's treatment of *arithmoi* and the accounts of odd and even is compatible with their definitions found in Euclid. Accordingly, I have pointed to other passages in Plato's dialogues that are in accord with the definitions and postulates in Euclid's Elements.

4 Cf. Kanayama 2000, 74, 'he [Socrates] grounds the safety of the subtler answers on the safety of the old safe answers.'

5 E.g. Arist. *Metaph.* A 6, 987b15, 991b217-31, 992b16 and 995b17, Z 1028b18-21, K 1069a34-6,  $\lambda$  1076a19-21, M 1090a4-6, N 1090b32-1091a5, and also, see Plato *Philebus* 16d7-e2.

6 Pritchard 1995, 14.

7 Burger 1984, 261n.9.

8 Cf. Burnet 1892, repr. 2018, 313, n. 42, 'The use of the zero was unknown in antiquity, and this made all modern arithmetical methods impossible.'

9 For example, Gallop 1993, 97, Schiller 1967, 57, Bluck 1957, 119, Haynes 1964, 18, Rist 1964, 29-30 claim there is no distinction.

10 In the *Theatetus* at 198d8-c10 Socrates discusses the knowledge of number, as it applies to  $\grave{\alpha}\rho_i\theta_\mu oi$ in the soul and the external objects that possess them. In *Philebus* at 56d-e, Socrates distinguishes the counting that the ordinary arithmetician does, with unequal units, and the counting that the philosopher does, with equal units. See Lasserre 1966, 22-25.

11 Schiller 1967, 57 is the exception, who understands the difference between  $\tau \dot{\alpha} \tau \rho i \alpha$  and  $\dot{\eta} \tau \rho i \dot{\alpha} c$  as the number three, 'which is different from things (which it occupies) and threeness (which occupies it)'.

12 Burger 1984, 261n.9.

13 Burger 1984, 261 n.9.

14 A similar 'by nature' claim was established previously in the text at *Phaedo* 103c1-2.

15 Cf. Kanayama 2000, 52.

16 Greek texts are from J. Burnet, 1900 and 1901 and W.D. Ross, 1924. http://stephanus.tlg.uci.edu/Iris/ indiv/browser.jsp#doc=tlg&aid=0059&wid=004&q=PLA TO&st=0.

17 There is much debate in the literature regarding the status of these characteristics. They can be 'immanent Forms', 'Forms', or sensible qualities or characteristics.

18 We should understand 'body' here to be also a 'figure', where Socrates says in the *Meno* at 75a-76a that a figure has color. It follows that all bodies/figures are perceptible through sense perception. For discussion, see Heath 1981, 292-293.

19 Pritchard 1995, 150-151.

Klein 1968, 46-60; Pritchard 1995, 15-16, 63-78.
For example, three multiplied by three is a square *arithmos*, while three multiplied by four is an oblong *arithmos*.

22 Klein 1968, 57.

23 Contra Kanayama 2000, 82, who says that the unit, while never admitting the Form of the Even, is not odd since it is not a number. While it is true that the unit is not a number, and so not an odd number, it doesn't follow that it is not odd.

24 Euclid IX, proposition 27 says that, 'if from an odd number an even number be subtracted, the remainder will be odd.' Thus, it is obvious if one takes three, an odd number, and subtracts from it two, an even number, the remainder will be 'one', which is not a number, but is nevertheless, odd. I used Heath's translation of Euclid's Elements from 2012.

25 Burnet 1911, reprinted 1959, 105.

26 Kanayama 2000, 82.

27 Bluck 1955, 124.

28 Hackforth 1955, repr. 1991, 162.

29 Bluck 1955, 75.

30 Hackforth 1955, repr. 1991, 158n.2. He cites Stobaeus, *Ecl.* I, but we need not use Stobaeus, for  $\pi\epsilon\rho(\tau\tau \dot{\sigma}\varsigma$  is in Euclid's 7<sup>th</sup> definition in Book VII of his *Elements*.

31 Prince 2011, 22-27, at 27, 'Socrates's overall argument does not succeed'; Bostock 1989, 184-191, at 191, 'there is still a gap in the argument' i.e., that soul must be shown to be a proper cause of life; Keyt 1963, to name a few.

32 Bostock 1986, repr. 1989, 185. Bostock is referring to *Phd*. 104 d6, ή τῶν τριῶν ἰδέα, which could mean 'the Form of three' but it could also be 'the Form of three things', which would be περιττός.

33 Bostock, 1989, 188.

34 Schiller 1967, 51-58.

35 Cf. Philebus 56d4-57a4, where Socrates and Protarchus distinguish between two kinds of calculating and measuring, one practiced by merchants and builders, the other practiced by the philosophers. The main difference is that the first calculate and measure with unequal units,  $\mu$ ovάδας ἀνίσους (56d9-10) whereas the philosophers calculate and measure with an infinite many equal units, (56e2-3). Also see *Theaetetus* 198c1-2, where a man can count numbers alone, αὐτὸς πρὸς αὐτὸν αὐτὰ, or count that which has number.

36 Altman 2016, 377-378.

37 In fact, we could understand a body as simply a collection of unequal Form characteristics.

38 Schiller 1967, 53-57.

39 Hackforth 1955, 195-197, translates the objections of Strato which were noted in Olympiodorus's commentary on the *Phaedo*, no. B 'Objections to the Principle of Exclusion of Opposites.' Keyt 1962, 172, imputes to Plato a fallacy of composition.

40 Rowe 1993, 262.

41 Burnet 1911, 123.

42 Kanayama 2000, 97, says that Socrates leaves this principle, that whatever is ἀθάνατος is indestructible, unexamined. Hackforth says that really nothing more has been shown Hackforth 1955, 164. Williamson calls it 'logically worthless', Skemp calls this move 'a blatant *petitio principii*.' Williamson and Skemp are quoted in Bluck 1955, Appendix Nine, "The Proof of the Soul's Indestructibility," 188.

43 Bostock 1986, 192.

44 Pakaluk 2003, 92.

45 After 104e8, Socrates doesn't use the singular feminine of  $\dot{\eta}$  τριάς.